

MATH 520 (Section 001)
Prof. Meade

University of South Carolina
Fall 2011

Exam 2
October 17, 2011

Name: _____
SS # (last 4 digits): _____

Instructions:

1. There are a total of 6 problems on 2 pages. Check that your copy of the exam has all of the problems.
2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and should not involve unreasonable manual calculations.
3. Be sure you answer the questions that are asked.
4. You must show all of your work to receive full credit for a correct answer. Correct answers with no supporting work will be eligible for at most half-credit.
5. Your answers must be clearly labeled and written legibly on additional sheets of paper (that I will provide). Be sure each sheet contains your name and the work for each question is clearly labeled.
6. Check your work. If I see *clear evidence* that you checked your answer (when possible) and you *clearly indicate* that your answer is incorrect, you will be eligible for more points than if you had not checked your work.

| Problem | Points | Score |
|---------|--------|-------|
| 1 | 10 | |
| 2 | 15 | |
| 3 | 15 | |
| 4 | 20 | |
| 5 | 20 | |
| 6 | 20 | |
| Total | 100 | |

Have a Great Fall Break!

1. (10 points) Determine the longest interval in which the initial value problem

$$t(t - 4)y'' + 3ty' + 4y = 2, \quad y(-2) = 2, \quad y'(-2) = 1.$$

is certain to have a unique twice differentiable solution. *Do not attempt to find the solution.*

2. (15 points)

(a) Verify that the functions $y_1(x) = x$ and $y_2(x) = xe^x$ are solutions of

$$x^2y'' - x(x + 2)y' + (x + 2)y = 0, \quad x > 0.$$

(b) Do they constitute a fundamental set of solutions?

3. (15 points) Find the solution of the initial value problem

$$y'' + 2y' + 5y = 0, \quad y(0) = 2, y'(0) = -2.$$

4. (20 points) The function $y_1(t) = t^{-1}$ is a solution of the differential equation

$$t^2y'' + 3ty' + y = 0, \quad t > 0.$$

Use the method of reduction of order to find a second solution of this differential equation. What is the general solution to this differential equation?

5. (20 points) Find the general solution of the differential equation

$$y'' + 2y' = 3 + 4 \sin(2t).$$

6. (20 points) Use the method of variation of parameters to find a particular solution of the differential equation

$$y'' - 4y' + 4y = \frac{e^{2t}}{t}.$$