MATH 511, Meade HW Solutions
2.3 – 3, 5, 9, 15, 10, 16 2/6/04

3.
a. P $(A_1 \cap B_2)$ = 5/35
b. P $(A_1 \cup B_1)$ = 26/35
c. P $(A_1 \mid B_1)$ = 5/19
d. P $(B_2 \mid A_2)$ = 9/23
e. Left. Given that the person is left thumb on top, there is a 74% chance that they be right eye dominant, but there is only a 56% chance given that they are right thumb on top. $P(A_2 \mid B_1) = 14/19 > P(A_2 \mid B_2) = 9/16$

5. P(A) = 0.7, P(B)= 0.5, P[(A U B)'] = 0.1
a. P[(A U B)'] = 1 – P(A U B) = 1 – P(A) – P(B) + P(A ∩ B)
0.1 = 1 – 1.2 + P(A ∩ B)
P(A ∩ B) = 0.3
b. P(A | B) = P(A ∩ B) / P(B) = 0.3 / 0.5 = 3/5
c. P(B | A) = P(A ∩ B) / P(A) = 0.3 / 0.7 = 3/7

9. *NOTE: Read the problem carefully. The question does NOT say that the first ball is orange. The given event is that at least one orange ball is selected.*

An urn contains four balls: two are orange and two are blue. Two balls are selected at random w/o replacement. You are told that *at least one* of them is orange. What is the probability that the other is also orange?

Combinatorics Approach:

Let A = {at least one ball is orange}
Let B = {both balls are orange}
The question is asking what is P(B | A)?

How many ways can any two balls be selected? $_4C_2 = 6$

How many ways can *at least one* be selected? Either one blue and one orange are selected or two oranges and zero blues are selected.
$(_2C_1)(_2C_1) + (_2C_2)(_2C_0) = (2)(2) + (1)(1) = 4+1 = 5$, so P(A) = 5/6

How many ways can exactly two orange balls be selected? This means to select two orange and zero blue. $(_2C_2)(_2C_0) = (1)(1) = 1$, so P(B) = 1/6

P(B ∩ A) = 1/6, since B is a subset of A.

Finally, P(B | A) = P(B ∩ A)/ P(A) = (1/6) / (5/6) = 1/5

Another Approach:

Let A = {at least one ball is orange}
Let B = {both balls are orange}
Again, we need to find P(B | A).

A is the same as the complement of zero balls being orange or the complement that both are blue. P(A) = 1 – P({both are blue})

P({both are blue }) = P({the first is blue} ∩ {the second is blue})
= P({first is blue})P({the second is blue} | {first is blue})
= (1/2) (1/3) = 1/6
So, P(A) = 1- (1/6) = 5/6

P(B) can be found in similar way as the P({both are blue}). So, P(B) = 1/6

P(B ∩ A) is again 1/6, since B is a subset of A.

Finally, P(B | A) = P(B ∩ A) / P(A) = (1/6) / (5/6) = 1/5

15. Consider the birthdays of a the students in a class size of r. Assume that no one has a leap day birthday.
    a. How many different permutations of birthdays are possible with repetition? 365^r
    b. How many permutations of birthdays are without repetition? ${}_{365}P_r$
    c. What is the probability that at least two students have the same birthday? This is the complement of how many arrangements of birthdays don't have any repetitions.
    $(365^r - {}_{365}P_r) / 365^r$ or $1 - ({}_{365}P_r / 365^r)$
    d. 23, as seen from the table of probabilities at the end of this document. This table was created in Excel (the spreadsheet can be downloaded from the course website.

10. An urn contains 17 balls marked LOSE and 3 balls marked WIN. You and an opponent take turns randomly selecting a ball from the urn. The person who draws the third WIN ball wins regardless of who drew the first two WIN balls.

    a. If you draw first, what is the probability that you win on your second draw?

Let A ={ you draw W first}
Let B ={ opponent draws W first}
Let C ={ you draw W second}

$P(A \cap B \cap C)=P(A)\ P(B \mid A)\ P(C \mid (B \mid A))=(3/20)(2/19)(1/18)=1/1140$
Another way to do it is: $({}_2C_2)({}_1C_1) / ({}_{20}C_3) = 1/1140$

b. If you draw first, what is the probability that your opponent wins on his second draw?

Let A = { two W's are selected in the first three draws}
Let B = { a W is selected on the fourth draw}
$P(A \cap B) = P(A)\ P(B \mid A) = (({}_3C_2)\ ({}_{17}C_1) / ({}_{20}C_3))\ (1/17) = ({}_3C_2)/ ({}_{20}C_3)$
$= 3/1140$

c. If you draw first what is the probability that you win?
In general the probability of drawing first and winning on your n^{th} turn ($n>1$) is: $({}_{2(n-1)}C_2)/ ({}_{20}C_3)$

The sum from n = 2 to n=10 of the probabilities of each turn is your overall chance of winning.
$(1/ ({}_{20}C_3))\ (({}_2C_2)+({}_4C_2)+({}_6C_2)+\ldots+({}_{18}C_2)) = (1+6+15+\ldots+153)/1140 =$
$525/1140 = 46.1\%$

d. Is it best to go first or second? Second. Going second, you have a 53.9% chance of winning.

16. A bowl contains 17 red chips and one blue chip. In a class of 18 students, the one who selects the blue chip gets an A in the class.

a. If you had the choice of going first, fifth or last, which would it be? Why? It doesn't matter because your chance of winning will always equal 1/18.
P({winning on first}) = 1/18
P({winning on second}) = (17/18)(1/17) = 1/18
P({winning on third}) = (17/18)(16/17)(1/16) = 1/18, and so on
This is the same as if the chips were randomly passed out.

b. Would you change your mind if there were two blue chips and 16 red chips? No. It still doesn't matter, you chance of winning has double, but is the same regardless of when you choose to draw.

P({winning first}) = 2/18
P({winning second}) = (16/18)(2/17)+(2/18)(1/17) = (32+2)/(18*17)
= 34/(18*17) = 2/18, and so on…

Probability That at Least Two People in a Group of r People Have the Same Birthday							
r	Probability	r	Probability	r	Probability	r	Probability
1	0.0000	31	0.7305	61	0.9951	91	1.0000
2	0.0027	32	0.7533	62	0.9959	92	1.0000
3	0.0082	33	0.7750	63	0.9966	93	1.0000
4	0.0164	34	0.7953	64	0.9972	94	1.0000
5	0.0271	35	0.8144	65	0.9977	95	1.0000
6	0.0405	36	0.8322	66	0.9981	96	1.0000
7	0.0562	37	0.8487	67	0.9984	97	1.0000
8	0.0743	38	0.8641	68	0.9987	98	1.0000
9	0.0946	39	0.8782	69	0.9990	99	1.0000
10	0.1169	40	0.8912	70	0.9992	100	1.0000
11	0.1411	41	0.9032	71	0.9993	101	1.0000
12	0.1670	42	0.9140	72	0.9995	102	1.0000
13	0.1944	43	0.9239	73	0.9996	103	1.0000
14	0.2231	44	0.9329	74	0.9996	104	1.0000
15	0.2529	45	0.9410	75	0.9997	105	1.0000
16	0.2836	46	0.9483	76	0.9998	106	1.0000
17	0.3150	47	0.9548	77	0.9998	107	1.0000
18	0.3469	48	0.9606	78	0.9999	108	1.0000
19	0.3791	49	0.9658	79	0.9999	109	1.0000
20	0.4114	50	0.9704	80	0.9999	110	1.0000
21	0.4437	51	0.9744	81	0.9999	111	1.0000
22	0.4757	52	0.9780	82	0.9999	112	1.0000
23	0.5073	53	0.9811	83	1.0000	113	1.0000
24	0.5383	54	0.9839	84	1.0000	114	1.0000
25	0.5687	55	0.9863	85	1.0000	115	1.0000
26	0.5982	56	0.9883	86	1.0000	116	1.0000
27	0.6269	57	0.9901	87	1.0000	117	1.0000
28	0.6545	58	0.9917	88	1.0000	118	1.0000
29	0.6810	59	0.9930	89	1.0000	119	1.0000
30	0.7063	60	0.9941	90	1.0000	120	1.0000