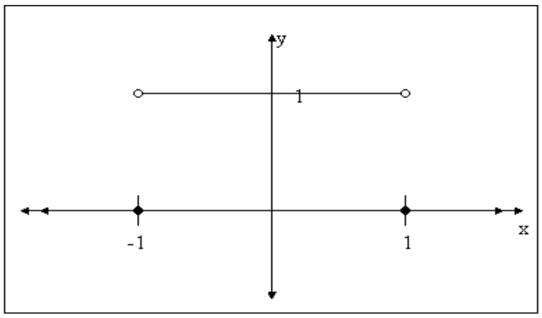
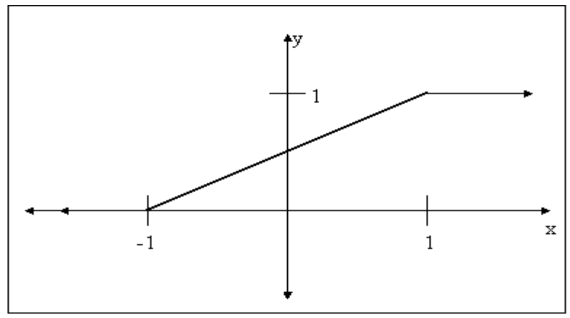
MATH 511, Meade HW Solutions
4.2 – 2, 3, 6abcd, 9, **5, **11 4/2/04

2. $\mu = (a + b)/2 = (-1 + 1)/2 =$ **0**
   $\sigma^2 = (b - a)^2/12 = (1 + 1)^2/12 = 4/12 =$ **1/3**

p.d.f



c.d.f



3. $X = U(0,10)$
   a. **$f(x) = 1/(b-a) = 1/10, \quad 0<x<10.$**
   Notice that since X is continuous it does not matter if the endpoints are included in the interval.
   b. $P(X \geq 8) = \int_8^{\infty}(1/10)dx = \int_8^{10}(1/10)dx =$ **0.2**
   c. $P(2 \leq X \leq 8) = \int_2^{8}(1/10)dx =$ **0.6**
   d. $E(x) = (0 + 10)/2 =$ **5**
   e. $Var(X) = (10)^2/12 = 100/12 =$ **25/3**

6. X has an exponential distribution such that $\theta = 20$. $f(x) = \underline{(1/20)}\, e^{-x/20}$
   a. $P(10 \leq X \leq 30) = (1/20)\int_{10}^{30} e^{-x/20}dx = - e^{-x/20}\,|_{(10,30)} = e^{-1/2} + e^{-3/2} =$ **0.3843**
   b. $P(X > 30) = (1/20)\int_{30}^{\infty} e^{-x/20}dx = \lim_{b\to\infty} (- e^{-x/20}\,|_{(b,30)}) = e^{-3/2} - e^{-\infty/2} = e^{-3/2}+0$
   $= e^{-3/2} =$ **0.2231**
   c. $P(X>40 \mid X>10) = P(X > 40)/P(X > 10) = e^{-40/20} / e^{-10/20} = e^{-3/2} =$ **0.2231**
   d. $Var(X) = \theta^2 = 20^2 =$ **400**
   $M(t) = 1/(1 - \theta t) = 1/(1 - 20t), \; t<1/20$

9.
   a. $M(t) = 1/(1 - 3t), \; t < 1/3$
   $f(x) = (1/3)\, e^{-x/3}, \; x > 0, \; \theta = 3$
   $\mu = \theta =$ **3**
   $\sigma^2 = \theta^2 = 3^2 =$ **9**

b. $M(t) = 3/(3 - t), \; t < 3$, divide by three on the top and bottom to get:
   $M(t) = 1/(1 - (1/3)t), \; t < 3$

   $f(x) = 3e^{-3x}, \; x > 0, \; \theta = 1/3$
   $\mu = \theta =$ **1/3**
   $\sigma^2 = \theta^2 = 3^2 =$ **1/9**

5. $Y = U(0,1)$
$W = a + (b - a)\,Y$, $a < b$

a. Find the (cumulative) distribution function of W.

$F(w) = P(W < w) = P(a + (b - a)Y < w)$
$= P((b - a)Y < w - a)$
$= P(Y < (w-a)/(b-a))$
$= G(\ Y = (w - a)/(b - a))$
$= (\ ((w - a)/(b - a)) - 0\ ) / (1-0)$
$= \mathbf{(w - a)/(b - a)\ ,\ a < w < b}$

b. How is W distirubted? **$U(a,b)$**

11. Let X have an exponential distribution such that $\theta > 0$. Show that:
$P(X > x+y \mid X>x) = P(X > y)$.

$P(X > x+y \mid X>x) = P(X > x+y) / P(X>x) = (1/\theta)\int_{x+y}^{\infty} e^{-t/\theta}dt \,/\, (1/\theta)\int_{x}^{\infty} e^{-t/\theta}dt$
$= e^{-(x+y)/\theta} \,/\, e^{-x/\theta} = e^{-y/\theta}$

and

$P(X > y) = (1/\theta)\int_{y}^{\infty} e^{-t/\theta}dt = e^{-y/\theta}$ □