MATH 511, Meade HW Solutions
3.2 –3 (omit c), 5, 14, 18, **3c, **8 2/13/04

3.

a. $f(x) = x/10$, $x = 1,2,3,4$
$E(x) = (1)(1/10) + (2)(2/10) + (3)(3/10) + (4)(4/10)$
$= (1 + 4 + 9 + 16)/10 = 30/10 =$ **3**

b. $f(x) = x/55$, $x = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$
$E(x) = \sum_{x=1}^{10} x^2/55 = [(10)(11)(21)/(6)]/55 = 385/55 =$ **7**

**c. $f(x) = 3(1/4)^x$, $x = 1, 2, 3, \ldots$
$E(x) = \sum_{x=1}^{\infty} 3x(1/4)^x = \sum_{x=1}^{\infty} 3x/4^x \leq \sum_{x=1}^{\infty} 3^x/4^x = 3$
This assures that the series does indeed converge. The series, $\sum_{x=1}^{\infty} x/4^x$, can now be approximated. After about ten approximations it is clear that the series converges to 0.44444… which is 4/9.
So the E(X) = 3(4/9) = **(4/3)**

d. $f(x) = (1/30)(x+1)^2$, $x = 0, 1, 2, 3$
$E(x) = (1/30)[0 + 4 + 18 + 48] = 70/30 =$ **7/3**

e. $f(x) = 2/n(n+1)x$, $x = 1, 2, 3, \ldots, n$
$E(x) = 2/n(n+1) \sum_{i=1}^{n} x^2 = [2/n(n+1)][n(n+1)(2n+1)/6] =$ **(2n+1)/3**

5. $E(x) = \dfrac{2{,}987{,}994(0) + 12{,}000(25) + 4(10{,}000) + 1(50{,}000) + 1(200{,}000)}{3{,}000{,}000} - 0.50$
= 19 ⅔ cents – 50 cents = **- 30⅓ cents**

**8. $f(x) = 6/(\pi^2 x^2)$, $x = 1, 2, 3, \ldots$
$E(x) = (6/\pi^2) \sum_{x=1}^{\infty} x/x^2 = (6/\pi^2) \sum_{x=1}^{\infty} 1/x$; this is the harmonic series, which diverges. Therefore E(x) does not exist. One way to show this is the integral test for infinite series.

14.

a. [16(25) + 3(100) + 1(300)]/20 = 1,000/20 = **50**. Note that this makes sense: 1000 students divided into 20 classes; the average class size must be 1000/20=50.

b. **f(x) = 0.4, x = 25**
= 0.3, x = 100
= 0.3, x = 300. Note that here you are selecting a random student, not a random class.

c. E(x) = 25(0.4) + 100(0.3) + 300(0.3) = **130**

18.

a. $E[(X - \mu)/\sigma] = (1/\sigma) E(X - \mu) = (1/\sigma)[E(X) - E(\mu)] = (1/\sigma)[\mu - \mu] =$ **0**

b. $E\{[(X - \mu)/\sigma]^2\} = (1/\sigma^2)(E[(X - \mu)^2]) = (1/\sigma^2)(E(X^2) - 2\mu E(X) + \mu^2)$
$= (1/\sigma^2)(E(X^2) - 2\mu^2 + \mu^2) = (1/\sigma^2)(E(X^2) - \mu^2)$
$= (1/\sigma^2)\sigma^2 =$ **1**

3.3 – 3, 5, 7(abc), 14, **7(de)

3. There is six-question multiple choice test with five possible answers, only one of which is correct. If a student guesses randomly and independently, then what is the probability that:
   a. The student gets questions 1 and 4 correct.
      $(0.2)^2(0.8)^4$ = **0.0164**
   b. The student gets exactly two questions right. ${}_6C_2\ (0.2)^2(0.8)^4$ = **0.2458**

5. The number of people who believe that the IRS abuses their power has a binomial distribution. b(25,0.7). In order to solve this problem using Appendix C, Table II, one needs to convert the problems so that $p \le 0.5$.
   So Let Y = the number who do not believe the IRS abuses their power. Y has a distribution of b(25, 0.3).

   a. $P(X \ge 13) = P(Y \le 12)$ = **0.9825**
   b. $P(X \le 11) = P(Y \ge 14) = 1 - P(Y < 14) = 1 - P(Y \le 13) = 1 - 0.9940$ = **0.0060**
   c. $P(X = 12) = {}_{25}C_{12}\ (0.7)^{12}(0.3)^{13}$ = **0.0115**
   d. $\mu(x) = np = (25)(0.7)$ = **17.5**
      $\sigma^2\ (x) = np(1 - p) = (25)(0.7)(0.3)$ = **5.25**
      $\sigma(x) = \sqrt{\sigma^2}$ = **2.29**

7.
   a. W has a binomial distribution **b(2000,** $\pi/4$). The probabilty comes from that the fact that exactly one fourth of the unit circle sits inside the unit square. The area of the unit circle is $A_{circle}(r = 1) = \pi(1)^2 = \pi$.
   b. $\mu(w) = np = (2000)(\pi/4)$ = **1,570.796**
      $\sigma^2\ (w) = np(1 - p) = (2000)(\pi/4)(1 - \pi/4)$= **337.096**
      $\sigma(w)= \sqrt{\sigma^2}$ = **18.360**
   c. $E(W/500) = \mu(w)/500 = \pi$
 **d. Click on the Excel link on the solutions column for a spreadsheet concerning parts d and e of this problem.
 **e. Notice that exactly one eighth of the unit sphere is contained in the unit cube The volume of the unit sphere is $(4/3)\ \pi$. So the probability that a point on The unit cube is also on the unit sphere is $(1/8)\ (4/3)\ \pi = \pi/6$

14.
   a. **X = b(8, 0.90)**, Let Y = number of mints weighing less than 20.7g = b(8, 0.10)
   b.
      i. $P(X = 8) = {}_8C_8\ (0.90)^8(0.10)^0$ = **0.4305**
      ii. $P(X \le 6) = P(Y \ge 2) = 1 - P(Y < 2) = 1 - P(Y \le 1) = 1 - 0.8131$ = **0.1869**
      iii. $P(X \ge 6) = P(Y \le 2$ = **0.9619**