3. P(A) = 1/4 , P(B) = 2/3 , A and B are independent

a. $P(A \cap B) = (1/4)(2/3) =$ **1/6**
b. $P(A \cap B') = (1/4)(1 - 2/3) = (1/4)(1/3) =$ **1/12**
c. $P(A' \cap B') = (1 - 1/4)(1 - 2/3) = (3/4)(1/3) =$ **1/4**
d. $P[(A \cup B)'] = 1 - P(A \cup B) = 1 - (P(A) + P(B) - P(A \cap B)$
$= 1 - (1/4) - (2/3) + (1/6) =$ **1/4**
e. $P(A' \cap B) = (1 - 1/4)(2/3) = (3/4)(2/3) =$ **1/2**

7. $P(A_1) = 0.5$, $P(A_2) = 0.7$, $P(A_3) = 0.6$, events are mutually independent

a. Compute the probability that exactly one player scores a field goal.

This means that one player succeeds while the other two fail. There are three ways that this can happen.

$P(A_1)\,P(A_2')\,P(A_3') + P(A_1')\,P(A_2)\,P(A_3') + P(A_1')\,P(A_2')\,P(A_3) =$
$(0.5)(0.3)(0.4) + (0.5)(0.7)(0.4) + (0.5)(0.3)(0.6) = 0.06 + 0.14 + 0.09 =$
0.29

b. Compute the probability that two are successful.

$P(A_1)\,P(A_2)\,P(A_3') + P(A_1)\,P(A_2')\,P(A_3) + P(A_1')\,P(A_2)\,P(A_3) =$
$(0.5)(0.7)(0.4) + (0.5)(0.3)(0.6) + (0.5)(0.7)(0.6) = 0.14 + 0.09 + 0.21 =$
0.44

17. Each of 12 students is assigned a number 1-12 and is given a 12-sided die

a. What is the probability of one at least match between what the students roll and their assigned number?

This is the complement of none of the students getting a match.
The probability that none of them gets a match is $(11/12)^{12}$ since each student has an 11/12 chance of not matching. So the probability of no one matching is: $1 - (11/12)^{12} =$ **64.8%**

b. If you are one of the students, what is the probability that at least one student matches the number that you roll?

The is the complement that no one matches you. The chance of someone not rolling the same number as you is 11/12. The chance that no one matches you is $(11/12)^{11}$ and the chance that at least one matches you is $1 - (11/12)^{11} =$ **61.6%**

10. Let D_1, D_2, D_3 all be four sided die. They are labeled as follows:
D_1: 0, 3, 3, 3 D_2: 2, 2, 2, 5 D_3: 1, 1, 4, 6

a. Find the probability that D_1 beats D_2. (3)(3)/ (4)(4) = **9/16**
b. Find the probability that D_2 beats D_3. [(3)(2) + (1)(3)]/ (4)(4) = **9/16**
c. Find the probability that D_3 beats D_1. [(2)(4) + (2)(1)]/ (4)(4) = **10/16**

16. You and an opponent take turns drawing from five balls. Four are marked LOSE and one is marked WIN. The first to draw the WIN ball wins. If you Draw first, what is you chance of winning if there is:

a. replacement? This means that the game goes on indefinitely and the the probability that any one wins on a particular draw is 1/5. The probability that the first person to draw wins is:

$1/5 + (4/5)^2(1/5) + (4/5)^4(1/5) + (4/5)^6(1/5) + (4/5)^8(1/5) + (4/5)^{10}(1/5) + \ldots$
$= (1/5)\ [\ 1 + (16/25)^1 + (16/25)^2 + (16/25)^3 + (16/25)^4 + (16/25)^5 + \ldots]$
$= (1/5)\ [\ 1/(1-(16/25))]$, since this is an infinite geometric series
$= (1/5)\ [1/(9/25)] = (1/5)\ (25/9) =$ **5/9**

b. no replacement? This means that there are just three ways for the first person to win. He wins on the first, third, or fifth draw. This problem is equivalent to 2.3-16a from last time. So, the probability of winning on a particular draw is always the same and is (1/5). The total probability that the first person wins is **3/5**.

2.5 – 2, 3, 6

2. Type A beans germinate 85% of the time and type B beans germinate 75% of the time. A bag of beans contains a mixture of these two types: 40% A and 60% B.

a. Find the probability that a randomly selected seed will grow.
$P(G) = P(G \cap A) \cup P(G \cap B) = P(A)\ P(G \mid A) + P(B)\ P(G \mid B)$
$= (0.4)(0.85) + (0.6)(0.75) = (0.34) + (0.45) =$ **0.79**
b. Given that a seed grows, what is the probability that is type A?
$P(A \mid G) = P(A \cap G) / P(G) = [P(A)\ P(G \mid A)] / P(G) = 0.34/ 0.79 =$ **0.43**

3.

a. Find the probability of selecting a Belgian coin

$P(B20) = P(B20 \cap C_1) \cup P(B20 \cap C_2)$
$= P(C_1)\ P(B20 \mid C_1) + P(C_2)\ P(B20 \mid C_2)$
$= 0.25\ (1/3) + 0.75\ (1/2) =$ **11/24**

b. Find the probability that the coin came from bag one given that it is a Belgian coin.

$P(C_1 \mid B20) = P(C_1 \cap B20) / P(B20) = [P(C_1) P(B20 \mid C_1)] / P(B20)$
$= [0.25 (1/3)] / (11/24) = \mathbf{2/11}$

6. Let A = the bag with 5 red and 20 yellow.
   Let B = the bag with 15 red and 10 yellow.

   a. $P(R) = P(R \cap A) \cup P(R \cap B) = P(A) P(R \mid A) + P(B) P(R \mid B)$
      $= (0.75) (0.2) + (0.25) (0.6) = 0.15 + 0.15 = \mathbf{0.30}$

   b. $P(Y) = P(Y \cap A) \cup P(Y \cap B) = P(A) P(Y \mid A) + P(B) P(Y \mid B)$
      $= (0.75) (0.8) + (0.25) (0.4) = 0.60 + 0.10 = \mathbf{0.70}$

   c. $P (B \mid R) = P(R \cap B) / P(R) = (B) P(R \mid B) / P(R) = (0.25) (0.6) / (0.30$
      $= \mathbf{1/2}$

3.1 – 3, 4, 6, 8

3.
   a. 1 + 2 + 3 + 4 = 10, c=**10**
   b. 1 + 2 + … + 10 = 55 , c=**1/55**
   c. $(1/4)^1 + (1/4)^2 + (1/4)^3 + (1/4) + \ldots + (1/4)^n = (1/4) [1/ (1 - (1/4)]$
      $= (1/4) (4/3) = 1/3$, c= **3**
   d. 1 + 4 + 9 + 16 = 30, c = **1/30**
   e. 1 + 2 + 3 + … + n = n(n+1)/2, c = **n(n+1)/2**

4.
   a. **f(x) = 1/10 ,   x = 0, 1, 2, …, 9**
   b.

x	freq.	rel. freq.
0	11	0.073
1	14	0.093
2	13	0.086
3	12	0.080
4	16	0.106
5	13	0.000
6	22	0.860
7	16	0.146
8	18	0.106
9	15	0.100

6.

a.

x	2	3	4	5	6	7	8	9	10	11	12
P(x)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

f(x) = (6 - |x-7|)/36 , x = 2, 3, …, 12

8.

a. **f(w) = 1/12,   x = 0,1, 2, …, 11**