Chapter 3 Homework Solutions

MATH 300 – Spring 2014

Prepared by Ben Aaron

2.

a) Key question: How do I show that an integer is even.
Answer: Show that the integer is twice another integer.
Apply to this problem: Show that n = 2*k for some integer k.

b) Key question: How do I show that an integer is prime?
Answer: Integer must be greater than 1 and only divisible only by 1 and itself
Apply to this problem: Show that n is both greater than 1 and only divisible by 1 and itself

4.

a) A1: From the definition of an odd integer, we gather that for n to be odd it must satisfy n = 2*k + 1, where k is an integer

b) A1: From the definition of rational numbers, we gather that for s and t to be rational numbers they must satisfy s = w/v , where w and v are integers and v ≠ 0, and t = x/y, where x and y are integers and y ≠ 0 and x ≠ 0.

5.

a) From the definition of a prime integer: 2^n -1 must be greater than 1 and only divisible by 1 and itself.

c) From the definition of a convex function, for all real numbers x & y, and all real numbers t with 0<=t<=1:
**(f + g) (t*x + (1-t)*y) ≤ t*(f + g)*x + (1-t) (f + g)*y**
**f*(t*x + (1-t) y) + g*(t*x + (1-t) y) ≤ t*(f(x) + g(x)) + (1-t) (f(y) + g(y)).**

d) From the definition of greater than or equal to functions: for every element x of the set S intersect T, f(x) ≥ g(x).

8. a)

| A | B | A ∨ B |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

b) ~A ∨ B is logically equivalent to A => B.

| A | B | ~A ∨ B |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

9.

b) If r is a real number such that $r^2 \neq 2$, then r is rational.

c) If quadrilateral ABCD is a rectangle, then it is a parallelogram with one right angle.

14.

a) If A implies B, B implies C, C implies D, and D implies A, then A is equivalent to B, C and D because they mutually imply one another, which fulfills the definition of equivalency. For example, it is given that A implies B, but it is also true that B implies A because B implies C, which implies D, which in turn implies A, and by the transitive property this means that B implies A.

b) If one wanted to prove the equivalency of A, B, C and D, the above method would be the quickest because only four proofs would be required, whereas if one were to prove the equivalency of each pair separately six proofs would be required, one each for the forward and backward implicates of A to B, A to C, and A to D. (Note added by Dr. Meade: Proving equivalency of A, B, C, and D requires more than showing A is equivalent to B, A to C, and A to D. It also requires showing B is equivalent to C, B to D, and C to D. So, actually there are 12 implications involved in showing A, B, C, and D are equivalent, but these can all be derived by using the four implications: A => B, B => C, C => D, and D => A.)

16. From proposition three, we gather that a right triangle ABC with sides of length a and b and hypotenuse of length c, and with $c = \sqrt{(2*a*b)}$, is an isosceles triangle.

20. In the sentence starting "Thus," this proof makes use of the transitive property to substitute in a $\sqrt{u/2v}$ for sin(U) in the equation sin(U) = u /w, therefore making $\sqrt{u/2v}$ = u /w. In the next sentence, beginning "By algebra," the omitted math is as follows:

| | |
|---|---|
| $w * \sqrt{u/2v} = u / w * w$ | Multiply both sides by w. |
| $w*\sqrt{u/2v} * \sqrt{2v/u} = u*\sqrt{2v/u}$ | Divide both sides by $\sqrt{u/2v}$ (positive). |
| $w = \sqrt{2uv}$ | Simplify. |

By this math one can see that the proof justifies the proposition.

24. From proposition 2 on page 27, we gather that any even integer n has a square, $n^2$, that is also an even integer. We know that the sum of two odd integers is an even integer, and so the sum of a and b, both odd integers, would have to be an even integer. Thus the square of (a + b), an even integer, by proposition 2 would have to be an even integer as well. Therefore $(a + b)^2$ is an even integer. Q.E.D.