Math 300
Chapter 10 Homework Solutions

1- a) Work forward from “n is odd” ; work backward from “n^2 is odd”
b) work forward from “T is bounded”, backward from “S is bounded”

2- a) work forward form “$n>p$”, backward from “n does not divide p”

3- No because Bob’s claim was that $A \Rightarrow B$ but Mary claimed that $B \Rightarrow A$ which does not hold true.
4- B; work forward from the assumption that there is a t with $0 < t < \pi/4$, such that $\sin(t) = r\cos(t)$. B results from squaring both sides the replacing $\cos^2(t)$ with $(1-\sin^2(t))$.

5-C

7- D is correct because the key question should refer to the conclusion, and when using the contrapositive method, the conclusion is the NOT of A.

A is incorrect because it refers to the NOT of B and is too specific
B is incorrect because it is too specific.
C is incorrect because it refer the the NOT of B.

8- D is correct because the key question should refer to the conclusion, and when using the contrapositive method, the conclusion is the NOT of A.

A is incorrect because it refers to B
B is incorrect because it is too specific.
C is incorrect because it refer the the NOT of B.

9 - The contradiction method is used in this proof because the author works forward from A and NOT B to reach the contradiction that the integer $p > p$.

15- Assume $p=q$. Therefore $\sqrt{pq} = \sqrt{p^2} = p = (2p)/2 = (p+p)/2 = (p+q)/2$. By the contrapositive method, since $\sqrt{pq} = (p+q)/2$, the proof is complete.

16- By contrapositive, assume $(a+b)/2 \le \sqrt{ab}$ to show $a=b$. Work forward form $(a+b)/2 \le \sqrt{ab}$ to get $(a+b)^2 \le 4ab$, under the conditions that $a+b$ and $2\sqrt{ab}$ are both greater than 0, which is true because a and b are both greater than 0. Then $a^2 + 2ab + b^2 \le 4ab$, subtract 4ab from both sides to get $a^2 - 2ab + b^2 \le 0$; so $(a-b)^2 \le 0$. Because a square can not be less than 0, $(a-b)^2 = 0$. Therefore, $a-b = 0$ and $a=b$.Since we've shown that $(a+b)/2 \le \sqrt{ab}$ implies $a = b$, the contrapositive implication is also true, namely: a does not equal b implies $(a+b)/2 \ge \sqrt{ab}$)."