Chapter 1 Solutions
Math 300 – Spring 2014
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1. (a) $ax^2 + bx + c = 0$, (c) Triangle XYZ is similar to triangle RST, and (e) For every angle t, $\sin^2(t) + \cos^2(t) = 1$ are all mathematical statements; (b) and (d) are only expressions.

2. (a) There is an even integer n that, when divided by 2, is odd, (c) If x is a positive real number, then $\log_{10}x > 0$, and (d) $\sin(\pi/2) < \sin(\pi/4)$ are mathematical statements.

3.

a. Hypothesis: The right triangle XYZ with sides of lengths $x + y$ and hypotenuse of length z has an area of $z^2/4$.
   Conclusion: Triangle XYZ is isosceles.

b. Hypothesis: n is an even integer.
   Conclusion: n^2 is an even integer.

c. Hypothesis: a, b, c, d, e, and f are real numbers, and $ad - bc \neq 0$.
   Conclusion: $ax + by = e$ and $cx + dy = f$ can be solved for x and y.

4.

a. Hypothesis: r is a real number that satisfies $r^2 = 2$.
   Conclusion: r is irrational.

b. Hypothesis: p and q are positive real numbers with $\sqrt{pq} \neq \frac{p+q}{2}$.
   Conclusion: $p \neq q$.

c. Hypothesis: x is a real number, $0 \leq x \leq 1$, and $f(x) = 2^{-x}$.
   Conclusion: $f(x) = x$ for some $0 \leq x \leq 1$.

8. Yes, Jack did not lose the contest, because the statement is true, meaning that if the hypothesis is true, the conclusion must also be. We know the hypothesis is true because it is impossible to be older than one's father, so Jack is younger. From that, we know he did not lose the contest – but, if a tie is possible, "not losing" is different from "winning".

10.

a. If $2 < 7$, then $1 < 3$.
   True, because A and B are true.

b. If $x = 3$, then $1 > 2$.
   False if A is true and B is false (T => F is a false statement)
   True if A is false and B is false (F => F is a true statement)

15.

B	A	B implies A
T	T	T
F	T	T
T	F	F
F	F	T

B => A is false only when the hypothesis is true and the conclusion is false. But, in spite of this, it is apparent that A=>B and B=>A are not logically equivalent.

16. To show that A implies B is false, one would want to find a case where A would be true while B is false.

17.

a. If $x > 0$, then $\log_{10}x > 0$.
When $0 < x < 1$, $\log_{10}x < 0$, so for any value of x between 0 and 1 the statement is false. So, the overall statement is false.

b. If n is a positive integer, then $n^3 \geq n!$
$10^3 = 1000$, while $10! = 3{,}628{,}8000$. In this case, $n = 10 > 0$, so the hypothesis is true.
$n^3 \geq n!$, however, is false, so the conclusion is false. The smallest counterexample is n=6.