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[**review.mws**

Solutions to Calculus Review Problems

Math 242 (sections 1 and 3) -- Spring 1998

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```
[ >
[ > restart;
[ > with( plots ):
[ > with( DEtools ):
[ >
```

1 a

The Maple commands for differentiation are diff and Diff. Both are used in the same way; the difference is that diff automatically evaluates the derivative while Diff leaves the derivative unevaluated. Here you can see the difference:

```
[ > prob1a := Diff( sin( Pi*r ) * exp( -r ), r );
[ > value( prob1a );
[ >
[ > diff( sin( Pi*r ) * exp( -r ), r );
```

In general it is better to use Diff as this allows you to verify that the problem has been entered correctly.

```
[ >
```

1 b

We approach this problem in the same way, except that we might give a separate name to the expression to be differentiated, *e.g.*, `expr` :

```
[ > expr := ln( (t*exp(t))^(1/3) );
[ > prob1b := Diff( expr, t );
[ > value( prob1b );
```

It is easy to see a common factor in the numerator and denominator of this result. To bring this to Maple's attention we can ask Maple to simplify its result :

```
[ > simplify( " );
[ >
```

Recall that we performed several simplifications to the expression prior to doing any differentiation. If we ask Maple to simplify the expression, it responds with :

```
[ > simplify( expr );
```

which was an intermediate step in our manual simplificiations. It is not reasonable to expect the computer to simplify expressions exactly as you and I would. In many instances you may need to do some manual manipulations to make Maple's answers look exactly like your own.

[>

2 a

Integration is very similar to differentiation. The two commands are int and Int - you are advised to use int in general.

```
> prob2a := Int( cos( x/4 ) + 6/x^3, x );
> value( prob2a );
```

[>

2 b

Notice that Maple did not include the constant of integration in the previous result - it is the user's responsibility to include this constant in an appropriate form when necessary. This example illustrates how this might be done :

```
> prob2b := Int( tan( 3*z ), z );
> value( prob2b ) + C;
```

[>

2 c

```
> prob2c := Int( x^2 * ln( x ), x );
> value( prob2c ) + C;
```

[>

2 d

Definite integrals are specified by including the interval of integration in the third argument of the `Int` command :

```
> prob2d := Int( r^2 * exp( -r^3 ), r=0..infinity );
> value( prob2d );
```

[>

2 e

```
> prob2e := Int( (t+14)/(t^2+3*t-4), t );
> value( prob2e );
```

[>

Going Further

[>

3

Let's begin by defining and plotting the function involved in this problem :

```
> f := 1/(16+w^2);
> plot( f, w=0..4, 0..0.1, title=`Function for problem 3` );
```

[>

The average value of f over the interval [0, 4] is given by the following expression :

```
> avgval := 1/4 * Int( f, w=0..4 );
```

The value of this expression is :

```
> f_avg := value( avgval );
```

To obtain an approximate numeric value for the average value the evalf command can be used :

```
> evalf( f_avg );
>
```

Going Further

4

This problem requires some thought and advance planning. In fact, the calculations are simple enough that manual calculations are the most efficient means of working this problem. The following approach is included simply to demonstrate the steps that would be used in a Maple solution to this problem.

The key is to obtain a formula for the straight line in the graph. (Be sure you see where this comes from!)

```
> lng := 0.7 + 0.3*x;
```

As a check, let's try to duplicate the plot in the handout :

```
> plot( lng, x=0..3, 0..1.6, title='Graph of ln( g(x) )' );
>
```

At present we know that $\ln(g(x)) = .7 + .3\,x$ and need to solve this equation for $g(x)$. There are a number of ways to do this, but one is to notice that $g(x) = \mathbf{e}^{\ln(g(x))}$, so that

```
> g := exp( lng );
```

And, finally, to force Maple to simplify this result we need to use the expand command (check that simplify does not yield the same results)

```
> g:= expand( g );

> plot( g , x = 0 .. 3, title ='Graph of g(x)');

>
```

This explains why "logarithmic plots" are used: all exponential plots look more or less the same, and it is certainly difficult to read
the coefficient 2.01375 and the growth rate 0.3 off the graph. But the equivalent values of 0.7 and 0.3 were easy to get off the straight line plot of ln(g(x)).

5

We know that the area between two curves is given by the (definite) integral of the height of the region defined by the two curves. To determine this region it is first necessary to find the point(s) where the two functions cross, *i.e.*, to find all solutions to the equation $2\,x^3 = -x^2 + 4\,x + 3$ Here is one way to do this :

```
> y1 := 2 * x^3;
> y2 := -x^2 + 4*x + 3;
> solve( y1 = y2, x );
>
```

To check these results and to get a picture of the region whose area we will find, plot the functions on an interval containing all of the points of intersection :

```
> plot( [ y1, y2 ], x=-2..2, title = 'Plot for # 5' );
>
```

The area of the region bounded by these two functions is

```
> area := Int( y2 - y1, x = -1 .. 3/2 );
> area = value( area );
> area = evalf( area );
>
```

8

All that is needed to answer this problem is a graph of the integrand :

```
> plot( sec(x)^3, x=0..Pi/6 );
```

In particular, since the integrand is increasing on the interval $\left[0, \frac{\pi}{6}\right]$, the Riemann sum formed using the right endpoints must be an overestimate of the actual value of the integral. In fact, the actual value of the integral is found to be

```
> Int( sec(x)^3, x=0..Pi/6 );
> value( " );
> evalf( " );
```

which is smaller than the value given in the problem. You can use Maple yourself, by the way, to compute the left- right- or middle-Riemann sums as follows.

```
> with(student):
> rightbox( sec(x)^3, x=0..Pi/6, 3);
> rightsum( sec(x)^3, x=0..Pi/6, 3); evalf(");
> rightsum( sec(x)^3, x=0 .. Pi/6, 500); evalf(");
>
```

9

This problem is not difficult to solve using Maple (or by hand). Note that the Maple solution utilized the piecewise command - you may not want to take the time to learn about this command at this point in the semester.

If you are still reading, it is necessary to define an expression that gives the number of staff people working at any hour of the day. The scheduled staffing is given by :

```
> staff := piecewise( t <= 1, 1, t<=7, 0, t <= 14, 3, t <= 16, 2, t <=
  22, 4, 1 );
>
```

The actual staffing is a little different - because the store is unmanned between 2pm and 4pm. Here is how the actual staffing function would look:

```
> staff2 := piecewise( t <= 1, 1, t<=7, 0, t <= 14, 3, t <= 15, 2, t
  <= 16, 0, t <= 22, 4, 1 );
>
```

Graphing the two staffing expressions in a single plot is not very interesting - except between t=14 (2pm) and t=16 (4pm).

```
> plot( [ staff, staff2 ], t=0 .. 24 );
>
```

The average number of staff on duty throughout the day can be computed in exactly the same way as in # 3 - the only difference is that the function is now discontinuous.

```
> avg := 1/24 * Int( staff2, t=0..24 );
> value( avg );
> evalf( " );
```

Note that the average number of staff on duty is NOT found by averaging the number of staff on duty during the different shifts.

```
>
>
```