\>

BDH2-6.mws

This worksheet is designed to be used in conjunction with Section 2.6 of the text. In particular, note how the nullclines are plotted and how this information is used to determine the different types of behavior that a system of ODEs can possess.

\>

## Getting started

## Defining the model

All differential equation models begin with a differential equation. If the problem is an intial value problem, an initial condition is also needed. Replace the question marks ( ? ) in the following input regions to define the relevant ODE model.

```
> ode1 := diff( x(t), t ) = 2*x(t)*(1-x(t)/2) - x(t)*y(t) ;
> ode2 := diff( y(t), t ) = 3*y(t)*(1-y(t)/3) - 2*x(t)*y(t) ;
> MODEL := { ode1, ode2 } :
```

$$ode1 := \frac{\partial}{\partial t} x(t) = 2\, x(t) \left( 1 - \frac{1}{2} x(t) \right) - x(t)\, y(t)$$

$$ode2 := \frac{\partial}{\partial t} y(t) = 3\, y(t) \left( 1 - \frac{1}{3} y(t) \right) - 2\, x(t)\, y(t)$$

```
> VAR := { x(t), y(t) } ;
>
```

## Nullclines

It is difficult to give a general outline for the determination of nullclines. The key is to set each RHS equal to zero and find all curves on which the resulting equation is satisfied.

### x-nullclines

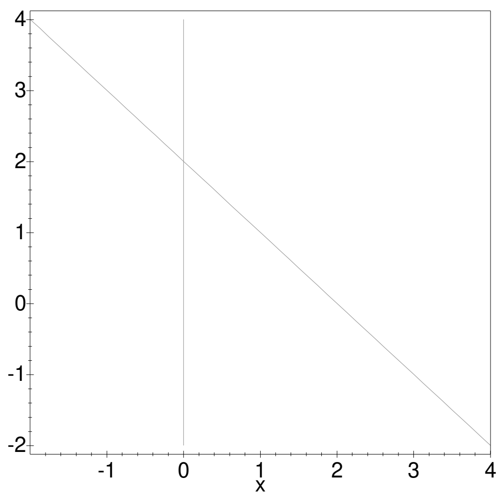
Here we illustrate first for the $x$-nullclines of the competing species example from the text:

```
> factor( rhs(ode1) );
```

$$-x(t)\,(-2 + x(t) + y(t))$$

Thus, the $x$-nullcline consists of the two lines $x = 0$ and $x + y = 2$. Here is how these might be plotted:

```
> xNULL1 := plot( [ 0, y, y=-2..4 ], color=GREEN ):
> xNULL2 := plot( 2-x, x=-2..4, color=GREEN ):
> xNULLplot := display( [ xNULL1, xNULL2 ], axes=BOXED ):
> xNULLplot;
```




```
>
```

### y-nullclines

And, now, the same general steps for the $y$-nullclines:

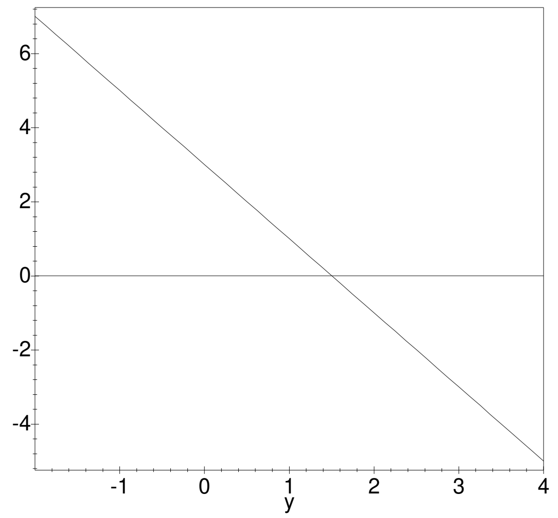
```
> factor( rhs(ode2) );
```

$$-y(t)\,(y(t) - 3 + 2\, x(t))$$

Thus, the $y$-nullcline consists of the two lines $y = 0$ and $2\,x + y = 3$. Here is how these might be plotted:

```
> yNULL1 := plot( 0, y=-2..4, color=BLUE ):
> yNULL2 := plot( 3-2*x, x=-2..4, color=BLUE ):
```

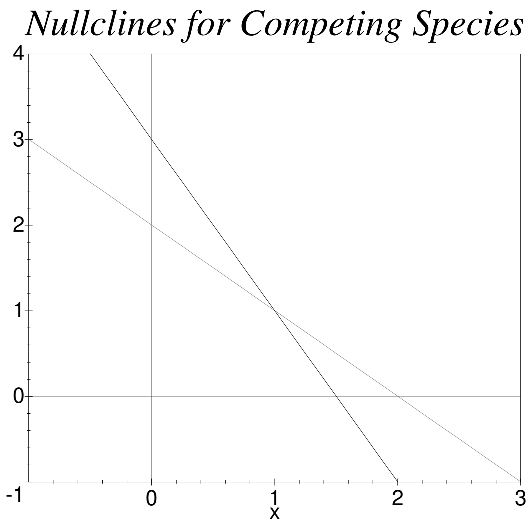
```
> yNULLplot := display( [ yNULL1, yNULL2 ], axes=BOXED ):
> yNULLplot;
```




```
>
```

The complete nullcline plot is the union of the plots of the $x$- and $y$-nullclines.

```
> NULLplot:=display( [xNULLplot, yNULLplot], view=[ -1 .. 3, -1 .. 4 ],
>                     title=`Nullclines for Competing Species` ):
> NULLplot;
```




Question

```
>
```

## Equilibrium Solutions

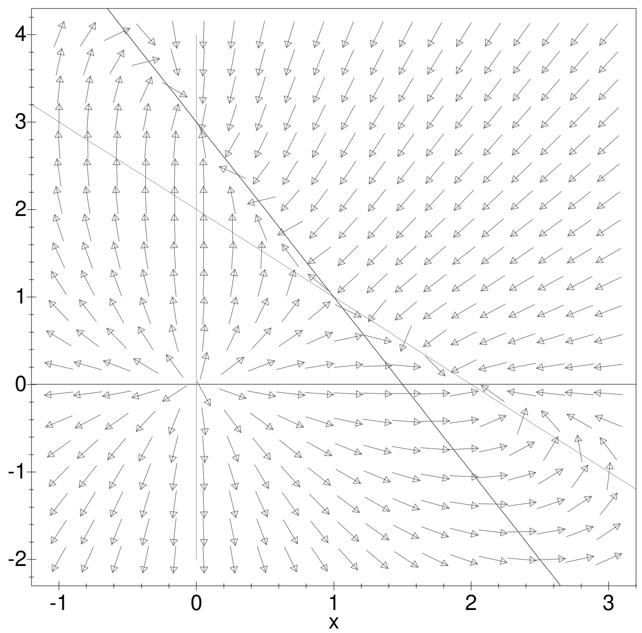
## Direction Field, Phase Portrait, and Solution Curves (DEplot)

```
> DOMAIN := t = 0 .. 5 ;
> RANGE := x = -1 .. 3, y = -2 .. 4 ;
>
```

### Direction Field

```
> DFplot := DEplot( MODEL, VAR, DOMAIN, RANGE, arrows = MEDIUM,
>                    title = `Direction Field for Competing Species` ):
> DFplot;
>
> display( [ NULLplot, DFplot ], title=`Nullclines and Direction Field` );
```

*Nullclines and Direction Field*

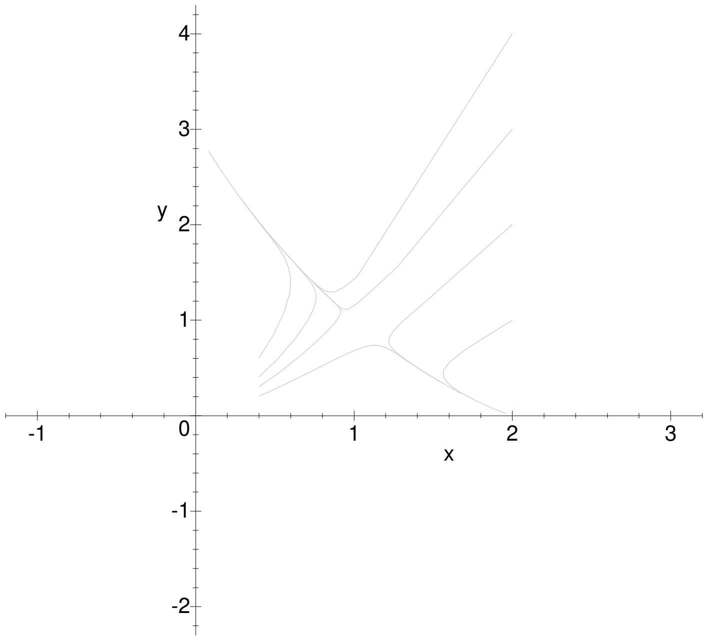



```
>
```

## Phase Portrait

```
> IC :=  [ x(0) = 0.4, y(0) = 0.2 ],
>        [ x(0) = 0.4, y(0) = 0.3 ],
>        [ x(0) = 0.4, y(0) = 0.4 ],
>        [ x(0) = 0.4, y(0) = 0.6 ],
>        [ x(0) = 2, y(0) = 1 ],
>        [ x(0) = 2, y(0) = 2 ],
>        [ x(0) = 2, y(0) = 3 ],
>        [ x(0) = 2, y(0) = 4 ] ;
>
> PPplot := DEplot( MODEL, VAR, DOMAIN, [ IC ], RANGE,
>                   scene=[ x, y ], arrows=NONE,
>                   title = `Phase Portrait for Competing Species` ):
> PPplot;
```

*Phase Portrait for Competing Species*



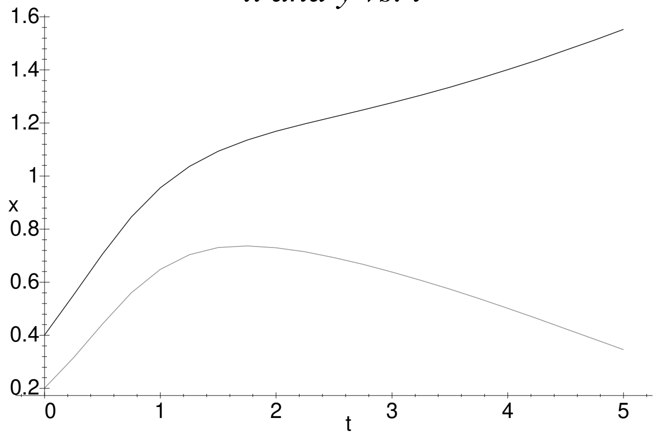

```
>
> display( [ DFplot, PPplot ], title=`Direction Field and Phase Portrait` );
> display( [ NULLplot, DFplot, PPplot ], title=`Nullclines, Direction Field, and Phase
  Portrait` );
>
```

## Solution Curves

Select the initial condition of your choice

```
> P1:= DEplot( MODEL, VAR, DOMAIN, [ IC[1] ],
>              scene=[ t, x ], arrows=NONE, linecolor=BLUE ):
> P2:= DEplot( MODEL, VAR, DOMAIN, [ IC[1] ],
>              scene=[ t, y ], arrows=NONE, linecolor=GREEN ):
> display( [ P1, P2 ] , title = `x and y vs. t` );     # combined solution curves
```

*x and y vs. t*




```
> #display( array([P1,P2]), title = `Solution Curve` ); # side-by-side solution curves
>
```

## Concluding Questions

```
>
> restart;
```