>

BDH1-3.mws

Slope Fields

Section 1.3 - Blanchard, Devaney, Hall

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Introduction

The purpose of this worksheet is to introduce you to the concept of slope fields (direction fields) of differential equations. This worksheet is intended to accompany reading section 1.3 of the text. The definitions are not repeated in this worksheet. There are a number of questions for you to answer as you work through this worksheet. Take the time to do what is asked, and be sure to ask questions as they arise.

Getting started

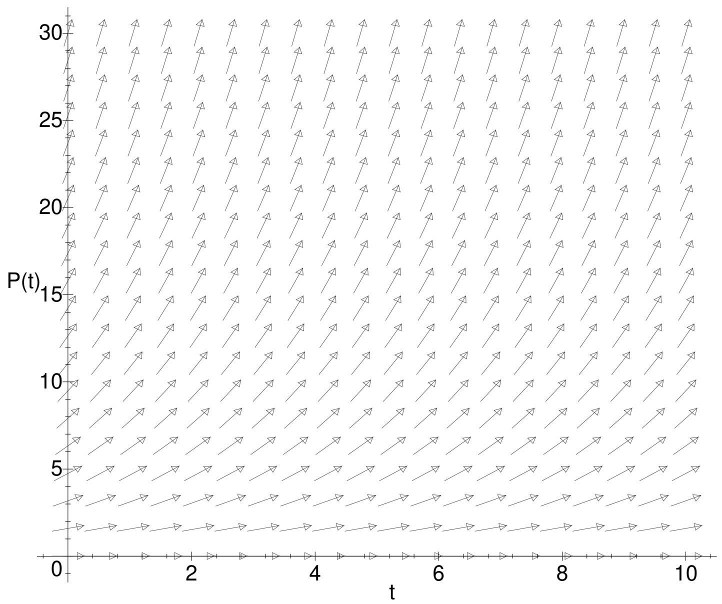
Unlimited population growth

A model for unlimited population growth is represented by the following first-order ordinary differential equation.

```
> MODEL1:= diff( P(t), t ) = 2/5 * P(t);
> variables:= { P(t) };
> IC1:= P(0) = 2;
> domain:= t = 0 .. 10;
>
```

The next command will exhibit the slope field for this model, and the following command superimposes the solution of the DE with the given initial condition.

```
> DEplot( MODEL1, variables, domain, P = 0 .. 30, arrows = MEDIUM );
```




```
> DEplot( MODEL1, variables, domain, P = 0 .. 30, [ [IC1] ], linecolor = BLUE, arrows =
  MEDIUM );
>
```

Logistic population growth

Let's compare the logistic model. Notice that to get a plot Maple needs to have specific numerical values for the parameters.

```
> MODEL2 := diff( P(t), t ) = 2/5 * ( 1 - P(t)/30 ) * P(t);
```

Let's use the same initial condition and domain as before.

```
> DEplot( MODEL2, variables, domain, P = 0 .. 40, [ [IC1] ], linecolor = BLUE, arrows =
  MEDIUM );
>
```

Questions

```
>
```

A non-autonomous model

Let's see what a non-autonomous model's slope field looks like.

```
> MODEL3:= diff( P(t), t) = exp( -0.25 * t );
> DEplot( MODEL3, variables, domain, P = 0 .. 15, arrows = MEDIUM );
> DEplot( MODEL3, variables, t = -2 .. 8, P = 0 .. 15, [ [IC1], [IC2], [IC3] ], linecolor =
  [BLUE, GREEN, BLACK], arrows = MEDIUM );
>
```

Questions

```
>
```

Testing Your Understanding - Mixing in a Vat

Replace the question marks (`?`) in the following input regions to create the slope fiield for the mixing problem discussed in section 1.2 of the text. Be sure to choose an appropriate range for the amount of sugar in the tank. Choose initial conditions that are both above and below the equilibrium solution.

```
> MODEL := diff( ?, ? ) = ? - ? ;   # define the differential equation model
> variables:= { ? };                # specify the variables in the model
> IC1:= S(0) = ?;                   # choose 4 initial conditions
> IC2:= S(0) = ?;
> IC3:= S(0) = ?;
> IC4:= S(0) = ?;
> domain:= t = 0 .. ?;             # specify a reasonable time interval
>
> DEplot( MODEL, variables, domain, S = ? .. ?, arrows = MEDIUM );
> DEplot( MODEL, variables, domain, S = ? .. ?, [ [IC1], [IC2], [IC3], [IC4] ], linecolor =
  [ BLUE, GREEN, BLACK, ORANGE ], arrows = MEDIUM );
>
```

Analytic solutions

All three of the models givcen here are solvable by separation of vartiables and explicit integration--so do this now by hand! Then use Maple to check your work. The Maple command for finding explicit solutions to differential equations is dsolve. *The arguments are the initial value problem (a set containing the differential equation and the initial condition) and the function for which we are trying to solve.*

```
> SOLN1 := dsolve( {MODEL1, IC1}, P(t) );
> SOLN3 := dsolve( {MODEL3, IC1}, P(t) );
> SOLN2 := dsolve( {MODEL2, IC1}, P(t) );
>
```

Problem 24, section 1-3 -- Piecewise-defined slope fields

```
>
> restart;
```