Math 241
Prof. Meade

University of South Carolina
Spring 2001

Final Exam
May 4, 2001

Name: ____________________
SS #: ____________________

Instructions:

1. There are a total of 9 problems on 10 pages. Check that your copy of the exam has all of the problems.
2. You must show all of your work to receive credit for a correct answer.
3. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	9	
2	9	
3	10	
4	12	
5	15	
6	9	
7	12	
8	12	
9	12	
Total	100	

Have A Great Summer!

1. (9 points) Let $\mathbf{a} = 5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - \mathbf{k}$, and $\mathbf{c} = \langle 1, 6, 0 \rangle$. Find each of the following:

   (a) $\mathbf{a} \cdot \mathbf{c}$

   (b) $\mathbf{b} \times \mathbf{c}$

   (c) $\mathbf{c} \cdot \mathbf{c} - |\mathbf{c}|^2$

2. (9 points)

   (a) What is the direction of the line $x = 2 - 3t$, $y = 3t$, $z = 2 - t$? What is a point on the line? NOTE: Be sure to clearly label the two answers.

   (b) Find parametric equations for the line through $(6, 1, -3)$ and $(-2, 1, 3)$.

   (c) Find an equation for the plane containing the lines $x = 3t$, $y = 1 + t$, $z = 4 + t$ and $x = 3 - 2s$, $y = 2$, $z = 5 + 4s$.

3. (10 points) Let C be the parametric curve $x = t^2$, $y = t^3$, $z = 3$.

   (a) Find all points on the curve with $x = 4$.

   (b) Find the tangent line to the curve at the point $(1, -1, 3)$.

   (c) Find the equation of the normal plane to the curve at the point $(1, -1, 3)$.

   (d) Find the speed of a particle that follows this curve as a function of t.

   (e) Find all points where the particle's speed is zero.

4. (12 points) Consider the curve $\mathbf{r}(t) = \langle t\cos t, t\sin t, 2t\rangle$, Find each of the following:

(a) $\mathbf{r}'(\frac{\pi}{2})$

(b) $\mathbf{T}(\frac{\pi}{2})$

(c) $\mathbf{r}''(\frac{\pi}{2})$

(d) a definite integral for the length of the curve between the points $(0,0,0)$ and $(-\pi, 0, 2\pi)$.
NOTE: Do *not* evaluate this integral.

5. (15 points) Let $f(x,y) = 3x^2y^4 + 7\frac{x^2}{y^3}$. Find

(a) $\nabla f(2,1)$

(b) f_{xx}

(c) $\dfrac{\partial^2 f}{\partial x \partial y}$

(d) the directional derivative of f at $(2,1)$ in the direction of $\mathbf{a} = \mathbf{i} - \mathbf{j}$

(e) $\dfrac{\partial f}{\partial t}$ where $x = e^t$ and $y = e^{-t}$
NOTE: Your answer should be expressed in terms of t.

6. (9 points) Find each limit or explain why it does not exist.

(a) $\lim_{(x,y)\to(0,0)} \dfrac{x^2 - y^2}{x^4 - y^4}$

(b) $\lim_{(x,y)\to(0,0)} \dfrac{x^4 - y^4}{x^2 + y^2}$

(c) $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2 + y^2}$

7. (12 points) Let $f(x, y) = x^2y - 6x^2 - 3y^2$.

(a) Find all critical points of f.

(b) Classify each of the critical points as a local maximum, local minimum, or saddle point, if possible.

8. (12 points)

(a) Evaluate $\int_1^2 \int_3^x \int_0^{\sqrt{3}y} \frac{z}{y^2+z^2}\, dz\, dy\, dx$.

(b) Sketch the domain of integration for the iterated integral $\int_0^1 \int_y^{\sqrt{y}} f(x,y)\, dx\, dy$.

(c) Interchange the order of integration in the integral in (b).

9. (12 points) Let S be the solid bounded below by $z = \sqrt{x^2+y^2}$ and above by the sphere $x^2+y^2+z^2=4$. The iterated triple integral for the volume of S is

$$V = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} dz\, dy\, dx.$$

(a) Show that the top and bottom surfaces intersect in the circle with radius $\sqrt{2}$ centered at the origin.

(b) Convert the triple integral for the volume of S to cylindrical coordinates.

(c) Convert the triple integral for the volume of S to spherical coordinates.

(d) Evaluate one of the three triple iterated integrals for the volume to show that the volume of S is $V = \frac{8}{3}(2 - \sqrt{2})\pi$.