MATH 241 University of South Carolina
Prof. Meade Spring 2001

Exam 2 Name: _____
April 26, 2001 SS #: _____

Instructions:

1. There are a total of 7 problems on 7 pages. Check that your copy of the exam has all of the problems.
2. You must show all of your work to receive credit for a correct answer.
3. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	10	
2	10	
3	20	
4	13	
5	25	
6	10	
7	12	
Total	100	

Good Luck!

1. (10 points)

   (a) In what direction is $f(x, y) = x^2 + y^4$ increasing most rapidly at the point $(-2, 1)$?

   (b) If $F(x, y) = x^3 - xy$, $x = 2\cos(3t)$, and $y = 3\sin(t)$, find $\dfrac{dF}{dt}$ at $t = 0$. (Express your answer as a function of t.)

2. (10 points) Find each limit or explain why it does not exist.

   (a) $\displaystyle\lim_{(x,y)\to(2,2)} \frac{x^2 - 2y}{x^2 + 2y}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - 4y^2}{x^2 + 2y^2}$

3. (20 points) For $f(x, y) = \frac{x^2}{2} + y^2$,

   (a) find the equation of the level curve that goes through the point $(4, 1)$;

   (b) find the gradient vector, ∇f, at $(4, 1)$;

   (c) draw the level curve and draw the gradient vector with its initial point at $(4, 1)$;

   (d) what special geometric relationship is there between the gradient vector at a point and the level curve through that point?

4. (13 points) Let $f(x, y) = x^2y - 6y^2 - 3x^2$.

   (a) Find all critical points of f.

   (b) Classify each of the critical points as a local maximum, local minimum, or saddle point, if possible.

5. (25 points)

(a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy \; dy \; dx$.

(b) Interchange the order of integration in the iterated integral $\int_0^1 \int_x^1 f(x,y) \; dy \; dx$.
HINT: Make a sketch first.

(c) Evaluate $\int_S \int \frac{1}{x^2+y^2} \; dA$ where S is the region between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

6. (10 points) Let S be the solid bounded by $z = x^2 + y^2$, $z = 0$, and $x^2 + (y-1)^2 = 1$.

   (a) Show that the cylinder $x^2 + (y-1)^2 = 1$ can be written as $r = 2\sin\theta$ in cylindrical coordinates.

   (b) Setup a triple iterated integral for the volume of S in cylindrical coordinates.

   (c) Evaluate the r and z integrals in the triple integral to show that the volume is $\displaystyle\int_0^{\pi} 4\sin^4\theta\, d\theta$. NOTE: The numerical value of this final integral is $\frac{3\pi}{2}$. Do *not* repeat this computation.

7. (12 points) Recall that the sphere of radius a is $x^2 + y^2 + z^2 = a^2$. Write the triple iterated integrals for the volume of a sphere of radius a in

   (a) Cartesian coordinates

   (b) cylindrical coordinates

   (c) spherical coordinates

   Which of these three integrals would you evaluate *if* you were asked to find the volume of a sphere of radius a? (Explain, but do *not* evaluate any integrals.)