

Exam 2
November 17, 2000

Name: _____
SS #: _____

Instructions:

1. There are a total of 6 problems on 6 pages. Check that your copy of the exam has all of the problems.
2. You must show all of your work to receive credit for a correct answer.
3. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	15	
2	10	
3	10	
4	20	
5	30	
6	15	
Total	100	

Good Luck!

1. (15 points) For each of the following limits, find the limit or explain why the limit does not exist.

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x-y}{x+y}$

(b) $\lim_{(x,y) \rightarrow (2,2)} \frac{x^2 - 2y}{x^2 + 2y}$

(c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + xy + y^2}{x^2 + y^2}$

2. (10 points) In what direction is $f(x, y) = x^2 + xy^4$ increasing most rapidly at $(1, -1)$?
Remember that a direction vector has unit length.

3. (10 points) Suppose $F(u, v) = \ln(uv)$, $u = \sqrt{xy}$, and $v = \sqrt{\frac{y}{x}}$. Use the Chain Rule to find $\frac{\partial F}{\partial y}$ in terms of x and y .
Be sure to simplify your answer.

4. (20 points)

(a) Formulate the constrained optimization problem for finding the largest possible rectangular box that is inscribed in the ellipsoid $36x^2 + 4y^2 + 9z^2 = 36$.

(b) Find the Lagrange equations for this problem.
Do not solve!

5. (30 points)

- (a) Evaluate $\int \int_S (x^2 + 2y) \, dA$ where S is the region between $y = x^2$ and $y = \sqrt{x}$.
(Sketch S .)

- (b) Convert $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} 2xy^2 \, dx \, dy$ to an equivalent integral in polar coordinates.
Do not evaluate the integral.

- (c) Rewrite the iterated integral $\int_0^2 \int_{x^2}^4 \int_0^{4-y} f(x, y, z) \, dz \, dy \, dx$ in the order $dx \, dy \, dz$.
HINT: A sketch should help.

6. (15 points) Find the Cartesian coordinate equation corresponding to

(a) the cylindrical coordinate equation $r^2 \cos(2\theta) + z^2 = 1$

(b) the spherical coordinate equation $\rho \sin \phi = 2$

HINT: Square the equation and use trigonometric identities.