MATH 241 (Section 502)
Prof. Meade

Exam 1
22 September 2010

University of South Carolina
Fall 2010

Name: Key
SS # (last 4 digits): ____

Instructions:

1. There are a total of 6 problems (not counting the Extra Credit problem).
2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and should not involve unreasonable manual calculations.
3. Be sure you answer the questions that are asked.
4. Copy your final answer to each question to the back of this page.
5. You must show all of your work to receive full credit for a correct answer. Correct answers with no supporting work will be eligible for at most half-credit.
6. Your answers must be clearly labeled and written legibly on additional sheets of paper (that I will provide). Be sure to put your name on each sheet, include the question number for all work, and staple all pages — in order — to this page when you turn in your completed test.
7. Check your work. If I see *clear evidence* that you checked your answer (when possible) and you *clearly indicate* that your answer is incorrect, you will be eligible for more points than if you had not checked your work.

Problem	Points	Score
1	25	
2	12	
3	15	
4	9	
5	15	
6	24	
Extra Credit	10	
Total	100	

Good Luck!

1. (25 points) Let $\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\mathbf{b} = \langle 3, -2, 1 \rangle$, and $\mathbf{c} = \mathbf{j} - 5\mathbf{k}$. Find

   (a) $\|\mathbf{b}\|$ $= \sqrt{14}$

   (b) $\mathbf{a} \times \mathbf{b}$ $= \langle -3, -7, -5 \rangle$

   (c) $\mathbf{c} \times \mathbf{c}$ $= \vec{0}$

   Note: $\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$.

   (d) $\mathbf{c} \cdot (\mathbf{b} \times \mathbf{a})$ $= -18$

   (e) $\mathbf{c} \times (\mathbf{b} \times \mathbf{a})$ $= \langle 40, -15, -3 \rangle$.

$\vec{v}_1 = \langle 4, -1, -1 \rangle$

$\vec{v}_2 = \langle 2, -1, 1 \rangle$

$\vec{n} = \vec{v}_1 \times \vec{v}_2$

2. (12 points) The points $(-1, 0, 2)$ and $(3, -1, 1)$ are on a plane $\mathcal{P}$. The plane $\mathcal{P}$ is also perpendicular to the plane $2x - y + z = 2010$. Find an equation for the plane $\mathcal{P}$.

   $\vec{n} = \langle -2, -6, -2 \rangle$ $\quad$ $\mathcal{P}: -2x - 6y - 2z = -2$.

3. (15 points) Find the point on the line with parametric equations $x = 2 - t$, $y = 1 + 3t$, $z = 4t$ that intersects the plane $2x - y + 2z = 2$.

   $2(2-t) - (1+3t) + 2(4t) = 2$

   $3t = -1$

   $t = -1/3$ $\quad$ $(7/3, 0, -4/3)$

4. (9 points) For each surface, classify it and indicate the axis:

   (a) $y = z^2$ parabolic cylinder, x-axis

   (b) $x^2 = y^2 + 4z^2$ elliptic cone, x-axis

   (c) $-4x^2 + y^2 - 4z^2 = 4$ hyperboloid of 2 sheets, y-axis

5. (15 points) Find the length of the curve $\mathbf{r}(t) = 2t^{3/2}\mathbf{i} + \cos(2t)\mathbf{j} + \sin(2t)\mathbf{k}$ for $0 \leq t \leq 1$.

   $L = \int_0^1 \sqrt{9t+4}\, dt = \frac{2}{27}(13^{3/2} - 8)$ $\quad$ $\vec{r}'(t) = 3t^{1/2}\vec{i} - 2\sin(2t)\vec{j} + 2\cos(2t)\vec{k}$

6. (24 points) For the curve given by $\mathbf{r}(t) = \langle t^2, \cos(t) + t\sin(t), \sin(t) - t\cos(t) \rangle$, find

   $\vec{r}'(t) = \langle 2t, t\cos t, t\sin t \rangle$

   $|\vec{r}'(t)| = (4t^2 + t^2)^{1/2} = \sqrt{5}\,t$

   (a) the unit tangent vector

   $\vec{T} = \frac{1}{\sqrt{5}} \langle 2, \cos t, \sin t \rangle$

   (b) the unit normal vector

   $\vec{N} = \langle 0, -\sin t, \cos t \rangle$

   $\vec{T}'(t) = \frac{1}{\sqrt{5}} \langle 0, -\sin(t), \cos(t) \rangle$

   (c) the curvature

   $\kappa = \frac{1}{5t}$

   $|\vec{T}'(t)| = \frac{1}{\sqrt{5}}(1)^{1/2} = \frac{1}{\sqrt{5}}$

   (d) the tangential component of acceleration

   $a_T = \sqrt{5}$.

   $\kappa = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}$ $\quad$ $a_T = v' = \frac{d}{dt}|\vec{r}'(t)|$

Extra Credit (10 points) A plane is flying east to west directly above the equator. In what direction is $\mathbf{B}$ pointing? Explain your reasoning.

HINT: The answer is one of North, South, East, West, Up, or Down.

$\vec{T}$: West

$\vec{N}$: Down

$\vec{B} = \vec{T} \times \vec{N}$: South