MATH 241 University of South Carolina
Prof. Meade Fall 2000

Exam 1 Name: _____
September 29, 2000 SS #: _____

Instructions:

1. There are a total of 7 problems on 6 pages. Check that your copy of the exam has all of the problems.
2. You must show all of your work to receive credit for a correct answer.
3. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	15	
2	21	
3	12	
4	12	
5	20	
6	10	
7	10	
Total	100	

Go Cocks! Unroll the 'Tide!

1. (15 points) Let $\mathbf{a} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, and $\mathbf{c} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$. Find each of the following:

   (a) $\mathbf{a} \cdot \mathbf{c}$

   (b) $\mathbf{b} \times \mathbf{c}$

   (c) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

2. (21 points)

(a) What is the direction of the line $x = 3 - 2t$, $y = t$, $z = -1$?

(b) Find parametric equations for the line through $(6, 1, -3)$ and $(-2, 2, -3)$.

(c) Find the center and radius of the sphere with equation $x^2 + y^2 + z^2 - 6x + 8y - 2z = 0$.

3. (12 points) Let $\mathbf{r}(t) = t\mathbf{i} + \frac{1}{2}t^2\mathbf{j} + \frac{1}{3}t^3\mathbf{k}$ for $-2 \leq t \leq 3$.

   (a) Find a vector equation for the tangent line to this curve at $t = 2$.

   (b) Find an equation for the normal plane to this curve at $t = 2$.

4. (12 points) Find the arclength of the curve $x = e^t \cos t$, $y = e^t$, $z = e^t \sin t$, for $1 \leq t \leq 5$.

5. (20 points) A particle is moving along a curve given by a vector-valued function $\mathbf{r}(t)$ for which $\mathbf{r}'(1) = 2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $\dfrac{d\mathbf{T}(1)}{dt} = -4\mathbf{i} + \mathbf{k}$. Find each of the following:

   (a) $\mathbf{T}(1)$

   (b) $\mathbf{N}(1)$

   (c) $\kappa(1)$

   (d) the unit binormal vector when $t = 1$

6. (10 points) Show that if the speed of a moving particle is constant, then its velocity and acceleration vectors are orthogonal.

7. (10 points) Sketch the level curves of $f(x, y) = (x + y^2)^2$ for $k = -1$, 0, 1, 2, 4.