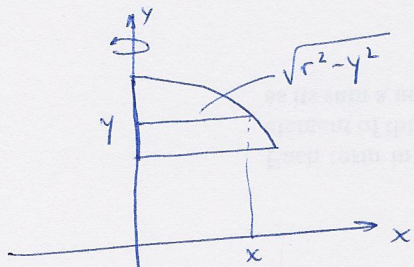
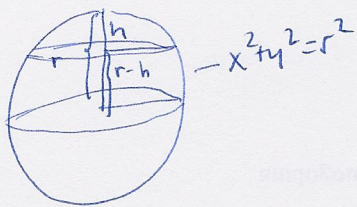


HW Soln for §6.2

#51.



$$R: 0 \leq x \leq \sqrt{r^2 - y^2} \\ h - r \leq y \leq r$$

$$V = \int_{r-h}^r \pi (\sqrt{r^2 - y^2})^2 dy$$

$$= \pi \int_{r-h}^r (r^2 - y^2) dy$$

$$= \pi \left(r^2 y - \frac{1}{3} y^3 \right) \Big|_{r-h}^r$$

$$= \pi \left(\left(r^3 - \frac{1}{3} r^3 \right) - \left(r^2(r-h) - \frac{1}{3}(r-h)^3 \right) \right)$$

$$= \pi \left(\frac{2}{3} r^3 - r^3 + r^2 h + \frac{1}{3} (r^3 - 3r^2 h + 3r h^2 - h^3) \right)$$

$$= \frac{\pi}{3} (3r h^2 - h^3)$$

$$= \pi h^2 \left(r - \frac{h}{3} \right)$$

Note: when $h=0$, $V=0$

when $h=r$, $V = \pi h^2 \left(r - \frac{h}{3} \right) = \pi r^2 \left(\frac{2r}{3} \right) = \frac{2\pi}{3} r^3$

↑ volume of a hemisphere
w/ radius r .