HW Solutions §11.2

#59. $\sum_{n=2}^{\infty} (1+c)^{-n}$ is a geometric series with ratio $r = \frac{1}{1+c}$ and first term $(1+c)^{-2}$.

$= \sum_{n=2}^{\infty} \left(\frac{1}{1+c}\right)^n$

When $|r| = \frac{1}{|1+c|} < 1$, it sums to $\frac{(1+c)^{-2}}{1-\frac{1}{1+c}} = \frac{(1+c)^{-2}(1+c)^2}{\left(1-\frac{1}{1+c}\right)(1+c)^2}$

$$= \frac{1}{(1+c)^2 - (1+c)} = \frac{1}{1+2c+c^2-1-c} = \frac{1}{c^2+c}$$

To have $\frac{1}{c^2+c} = 2$ requires $c^2+c = \frac{1}{2}$

$$c^2+c-\frac{1}{2} = 0$$

$$c = \frac{1}{2}\left(-1 \pm \sqrt{1+2}\right) = \frac{1}{2}(-1 \pm \sqrt{3}).$$

These values of c are approximately

$c = \frac{1}{2}(-1+\sqrt{3}) \approx 0.366$ $\qquad \frac{1}{1+c} \approx 0.732$

$c = \frac{1}{2}(-1-\sqrt{3}) \approx -1.366$ $\qquad \frac{1}{1+c} \approx -2.73$

Since $\left|\frac{1}{1+c}\right| = 2.73$ when $c = \frac{1}{2}(-1-\sqrt{3})$, this can't be a solution to our problem.

But, for $c = \frac{1}{2}(-1+\sqrt{3})$, $\frac{1}{1+c} \approx 0.732 < 1$ so this value of c is a solution to this problem.