HW Soln for §10.3

#69. $r = a\sin\theta + b\cos\theta$.

We prefer to see r^2, $r\cos\theta$, and $r\sin\theta$, so multiply this equation by r:

$$r^2 = ar\sin\theta + br\cos\theta.$$

$$x^2 + y^2 = ay + bx$$

Now complete the square – once in x & once in y:

$$x^2 - bx + y^2 - ay = 0$$

$$\left(x - \tfrac{b}{2}\right)^2 - \left(\tfrac{b}{2}\right)^2 + \left(y - \tfrac{a}{2}\right)^2 - \left(\tfrac{a}{2}\right)^2 = 0.$$

$$\left(x - \tfrac{b}{2}\right)^2 + \left(y - \tfrac{a}{2}\right)^2 = \frac{a^2}{4} + \frac{b^2}{4} = \frac{a^2+b^2}{4}.$$

This is the equation of a circle centered at $\left(\frac{b}{2}, \frac{a}{2}\right)$ with radius $\sqrt{\frac{a^2+b^2}{4}} = \frac{1}{2}\sqrt{a^2+b^2}$.