MATH 142 (Section H01)
Prof. Meade

Quiz 8
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University of South Carolina
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Name: Key
Section: H01

1. (4 points) Is the series $\sum_{n=1}^{\infty} \frac{n(n+5)}{(n+3)^2}$ convergent or divergent?

   Explain your answer. (If it converges, do we know how to find its sum?)

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n(n+5)}{(n+3)^2} = \lim_{n\to\infty} \frac{n^2+5n}{n^2+6n+9} = 1 \neq 0$$

Because $\lim_{n\to\infty} a_n \neq 0$, this series diverges by the n^{th} Term Test.

Note: If you try to apply the Integral Test you are faced with $\int a_n\,dn = \int \left(1 - \frac{1}{n+3} - \frac{6}{(n+3)^2}\right) dn = n - \ln(n+3) + \frac{6}{n+3}$ Ugh!!

2. (6 points) Consider the series $\sum_{n=1}^{\infty} \frac{x^n}{5^n}$.

   (a) Find the values of x for which the series converges.

$\sum_{n=1}^{\infty} \frac{x^n}{5^n} = \sum_{n=1}^{\infty} \left(\frac{x}{5}\right)^n$ is a geometric series with ratio $r = \frac{x}{5}$.

This series converges if & only if $|r| < 1$, so we need $\left|\frac{x}{5}\right| < 1$, or $|x| < 5$ or $-5 < x < 5$

   (b) Find the sum of the series that is valid for the values of x found in (a). (Simplify your answer so that it has no compound fractions.)

When $|x| < 5$, $\sum_{n=1}^{\infty} \frac{x^n}{5^n} = \frac{x/5}{1 - x/5} = \frac{x}{x-5}$.