MATH 142 (Section H01)
Prof. Meade

Quiz 2
September 9, 2014

University of South Carolina
Fall 2014

Name: Key
Section: H01

1. (5 points) Evaluate $\int_0^{\frac{\pi}{2}} \cos^5(y)\,dy$. $= \int_0^{\pi/2} \cos^4 y \cdot \cos y \, dy$

$$= \int_0^{\pi/2} \left(\cos^2 y\right)^2 \cos y \, dy$$

$$= \int_0^{\pi/2} \left(1-\sin^2 y\right)^2 \cos y \, dy$$

$u = \sin y$ $\begin{pmatrix} y=0 \Rightarrow u=0 \\ y=\pi/2 \Rightarrow u=1 \end{pmatrix}$

$du = \cos y \, dy$

$$= \int_0^1 \left(1-u^2\right)^2 du$$

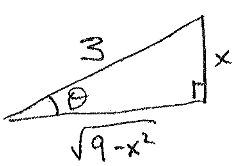
$$= \int_0^1 1-2u^2+u^4 \, du$$

$$= \left(u - \frac{2}{3}u^3 + \frac{1}{5}u^5\right)\Big|_0^1 = \left(1-\frac{2}{3}+\frac{1}{5}\right) - 0$$

$$= \frac{8}{15}$$

2. (5 points) Use the trigonometric substitution $x = 3\sin\theta$ to rewrite $\int x^3\sqrt{9-x^2}\,dx$ as an integral involving powers of trigonometric functions. **Do not evaluate this integral.**

$x = 3\sin\theta$



$$\int x^3\sqrt{9-x^2}\,dx = \int (3\sin\theta)^3\, 3\cos\theta\, (3\cos\theta)\,d\theta$$

$$= 3^5 \int \sin^3\theta \, \cos^2\theta \, d\theta.$$

$dx = 3\cos\theta\,d\theta$