MATH 142 (Section 502)
Prof. Meade

University of South Carolina
Fall 2008

Exam 3
November 6, 2008

Name: ______________________
Section 502

Instructions:

1. There are a total of 6 problems on 6 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	24	
2	24	
3	8	
4	8	
5	16	
6	20	
Total	90	

Do Your Best!

1. (24 points) [6 points each] Evaluate each definite integral.

   (a) $\displaystyle\int_0^\infty \frac{1}{x^{5/3}}\,dx$

   (b) $\displaystyle\int_{-\infty}^0 e^{-2x}\,dx$

   (c) $\displaystyle\int_2^\infty \frac{1}{x(\ln x)^2}\,dx$

   (d) $\displaystyle\int_{-1}^1 \frac{1}{1-x}\,dx$

2. (24 points) [6 points each] Determine if each sequence converges or diverges. If the sequence converges, find its limit.

(a) $\left\{ \dfrac{5^n}{2^{(n^2)}} \right\}_{n=1}^{\infty}$

(b) $\{\sin(2n)\}_{n=1}^{\infty}$

(c) $\dfrac{3}{2^2 - 1^2}, \dfrac{4}{3^2 - 2^2}, \dfrac{5}{4^2 - 3^2}, \ldots$

(d) $\dfrac{-1}{3}, \dfrac{2}{5}, \dfrac{-3}{7}, \dfrac{4}{9}, \ldots$

3. (8 points) Suppose the sequence $\{a_k\}$ is defined recursively by

$$a_0 = 1, \qquad \text{and} \qquad a_k = 1 + \sqrt{a_{k-1}} \quad \text{for } k \geq 1.$$

Assuming the sequence converges, find its limit.

4. (8 points) Show that the sequence

$$\left\{\frac{100^n}{n!}\right\}_{n=1}^{\infty}$$

is eventually strictly decreasing.

5. (24 points) [8 points each] Determine if each series converges or diverges. Do not attempt to find the sum of the series.

NOTE: Be sure to indicate the test used to reach your conclusion.

(a) $\displaystyle\sum_{k=1}^{\infty}\left(1+\frac{1}{k}\right)^{k}$

(b) $\displaystyle\sum_{k=1}^{\infty}k^{2}\left(\frac{2}{3}\right)^{k}$

(c) $\displaystyle\sum_{k=1}^{\infty}\frac{2^{k}}{k!}$

6. (20 points) [5 or 8 points each] For each of the following,

   (i) [2 points] rewrite as an infinite series in summation notation

   (ii) [3 points] determine if the series converges or diverges

   (iii) [3 points] if the series converges, determine the sum.

   (a) $\dfrac{3}{1 \cdot 2} + \dfrac{3}{2 \cdot 3} + \dfrac{3}{3 \cdot 4} + \dfrac{3}{4 \cdot 5} + \cdots$

   (b) $\ln(\frac{1}{2}) + \ln(\frac{2}{3}) + \ln(\frac{3}{4}) + \ln(\frac{4}{5}) + \cdots$

   (c) $0.14 + 0.0028 + 0.000056 + 0.00000112 + \cdots$

HINT: This is a geometric series.