

1. (5 points) Evaluate $\lim_{x \rightarrow \infty} \frac{5x+3}{x-4}$.

$$\begin{aligned} &= \lim_{x \rightarrow \infty} \frac{x(5 + \frac{3}{x})}{x(1 - \frac{4}{x})} \\ &= \lim_{x \rightarrow \infty} \frac{5 + \frac{3}{x}}{1 - \frac{4}{x}} \\ &= \frac{\lim_{x \rightarrow \infty} 5 + \frac{3}{x}}{\lim_{x \rightarrow \infty} 1 - \frac{4}{x}} = \frac{5}{1} = 5. \end{aligned}$$

2. (5 points) Explain why the function $f(x) = \begin{cases} e^x, & \text{if } x < 0 \\ x^2, & \text{if } x \geq 0 \end{cases}$ is discontinuous at $a = 0$.

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x^2 = 0$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} e^x = 1$$

since these are not equal, $\lim_{x \rightarrow 0} f(x)$

does not exist;

so f cannot be continuous at $x=0$.