MATH 141 (Sections 5 & 6)
Prof. Meade

University of South Carolina
Fall 2013

Exam 1
September 20, 2009

Name: Key
Section: 005 / 006 (circle one)

Instructions:

1. There are a total of 9 problems on 8 pages (front and back). Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

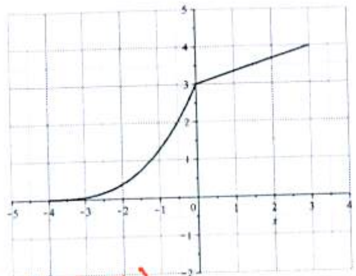
| Problem | Points | Score |
|---|---|---|
| 1 | 16 | |
| 2 | 18 | |
| 3 | 18 | |
| 4 | 15 | |
| 5 | 5 | |
| 6 | 6 | |
| 7 | 12 | |
| 8 | 5 | |
| 9 | 5 | |
| Total | 100 | |

Good Luck!

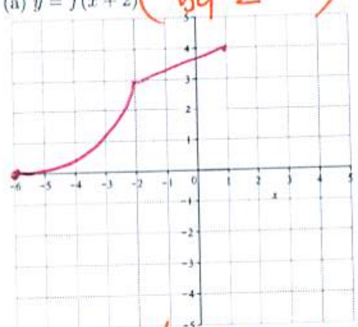


This page contains no test material.

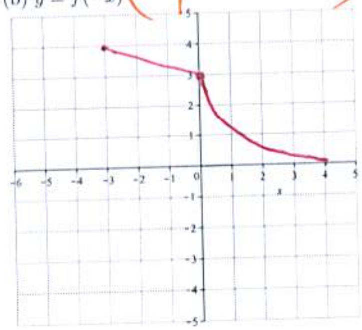
1. (16 points) The graph of $f$ is given. Draw the graphs of the following functions. Suppose the graph of $f$ is given.



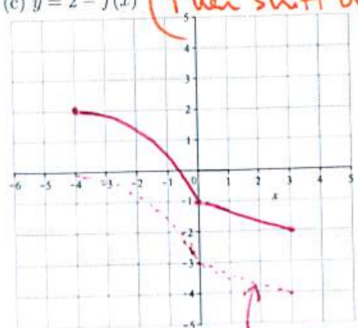
(a) $y = f(x+2)$ (shift left by 2)



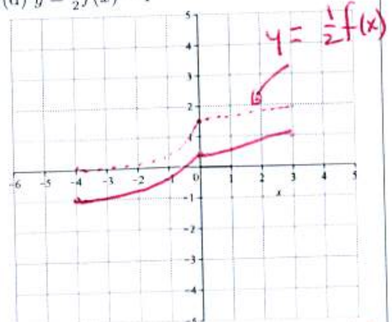
(b) $y = f(-x)$ (reflect about y-axis)



(c) $y = 2 - f(x)$ (reflect about x-axis then shift up by 2)




(d) $y = \frac{1}{2}f(x) - 1$




(scale by factor of 1/2 then shift down by 1)



2. (18 points)

(a) Find the exact value of $e^{3\ln(2)}$. $= e^{\ln(2^3)} = 2^3 = 8$

(b) Find the exact value of $\ln\left(\frac{1}{e^2}\right)$. $= \ln(e^{-2}) = -2$

(c) Solve the equation $e^{5-3x} - 10 = 0$.

$e^{5-3x} = 10$

$5 - 3x = \ln 10$

$-3x = \ln 10 - 5$

$x = \frac{\ln 10 - 5}{-3} = \frac{5 - \ln 10}{3}$

(d) Solve the equation $\ln(x) - \ln(x-1) = 1$.

$\ln\left(\frac{x}{x-1}\right) = 1$

$\frac{x}{x-1} = e^1 = e$

$x = e(x-1) = ex - e$

$x - ex = -e$

$(1-e)x = -e$

$x = \frac{-e}{1-e}$

$= \frac{e}{e-1}$.

(e) Find the exact value of $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \theta$

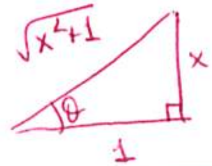
$\frac{\sqrt{3}}{2} = \sin\theta$

$\theta = \pi/3$ $(60°)$

(f) Simplify the expression $\cos(\arctan(x))$.

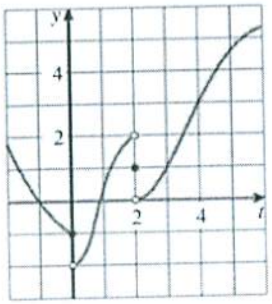
$\theta = \arctan x$

$\tan\theta = x$




$\cos(\arctan(x)) = \cos\theta = \frac{1}{\sqrt{x^2+1}}$.

3. (18 points) For the function $g$ whose graph is shown, state the value of each quantity (if it exists). If it does not exist, explain why.




(a) $\lim_{t\to 0^-} g(t) = -1$

(b) $\lim_{t\to 0^+} g(t) = -2$

(c) $g(0) = -1$

(d) $\lim_{t\to 2^-} g(t) = 2$

(e) $\lim_{t\to 2^+} g(t) = 0$

(f) $g(2) = 1$

(g) $\lim_{t\to 0} g(t)$ dne because $\lim_{t\to 0^-} g(t) \neq \lim_{t\to 0^+} g(t)$

(h) $\lim_{t\to 2} g(t)$ dne because $\lim_{t\to 2^-} g(t) \neq \lim_{t\to 2^+} g(t)$

(i) $\lim_{t\to 4} g(t) = 3$



4. (15 points) Evaluate each limit, if it exists. If a limit does not exist, explain why it does not exist.

(a) $\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \lim_{x\to 2} \frac{(x+3)(x-2)}{x-2} = \lim_{x\to 2} x+3 = 5$

(b) $\lim_{x\to -2} \frac{x^2-x+6}{x-2} = \frac{(-2)^2-(-2)+6}{-2-2} = \frac{12}{-4} = -3$

(c) $\lim_{t\to 0} \frac{1}{t} - \frac{1}{t^2+t} = \lim_{t\to 0} \frac{1}{t} - \frac{1}{t(t+1)} = \lim_{t\to 0} \frac{(t+1)-1}{t(t+1)}$

$= \lim_{t\to 0} \frac{\cancel{t}}{\cancel{t}(t+1)} = \lim_{t\to 0} \frac{1}{t+1} = \frac{1}{0+1} = 1.$

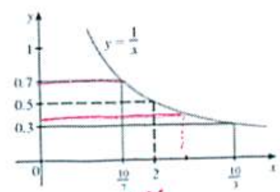
(d) $\lim_{u\to\infty} \frac{4u^4+5}{(u-2)^2(2u^2-1)}$

$= \lim_{u\to\infty} \frac{u^4(4+\frac{5}{u^4})}{u^4(1-\frac{2}{u})^2(2-\frac{1}{u^2})} = \lim_{u\to\infty} \frac{4+\frac{5}{u^4}}{(1-\frac{2}{u})^2(2-\frac{1}{u^2})} = \frac{4}{2} = 2$

(e) $\lim_{s\to\infty} \frac{s+2}{\sqrt{9s^3+3s^2+4s+1}}$

$= \lim_{s\to\infty} \frac{s(1+\frac{2}{s})}{s^{3/2}\sqrt{9+\frac{3}{s}+\frac{4}{s^2}+\frac{1}{s^3}}} = \lim_{s\to\infty} \frac{1+\frac{2}{s}}{s^{1/2}\sqrt{\ldots}} = 0$

5. (5 points) Use the given graph of $f(x) = 1/x$ to find a number $\delta$ such that if $|x-2| < \delta$ then $\left|\frac{1}{x} - 0.5\right| < 0.2$.




$2 - \frac{10}{7} = \frac{4}{7}$

$\frac{10}{3} - 2 = \frac{4}{3}$

$\delta = \frac{4}{7}$

immediately seen that $f$ is continuous at all $x \neq 0$ & $x \neq 2$

Checking $x=0$ & $x=2$:

6. (6 points) Let $f(x) = \begin{cases} 1+x^2 & \text{if } x \le 0 \\ 2-x & \text{if } 0 < x < 2 \\ (2-x)^2 & \text{if } x \ge 2 \end{cases}$.

Find the numbers at which $f$ is discontinuous.

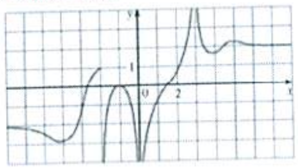
$f(0) = 1+0^2 = 1$

$f(2) = (2-2)^2 = 0.$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 1+x^2 = 1$ $\quad \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 2-x = 2$ $\quad \therefore \boxed{\text{not cont. @ } x=0.}$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 2-x = 0$ $\quad \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (2-x)^2 = 0$ $\quad \therefore$ cont. at $x=2$

7. (12 points) For the function $f$ whose graph is shown, state the value of each quantity (if it exists). If it does not exist, explain why.




(a) $\lim_{x\to\infty} f(x) = 2$

(b) $\lim_{x\to-\infty} f(x) = -2$

(c) $\lim_{x\to 3} f(x) = +\infty$

(d) $\lim_{x\to 0} f(x) = -\infty$

(e) The equation of each horizontal asymptote. $y=-2$ & $y=2$

(f) The equation of each vertical asymptote. $x=-2,\ x=0,$ & $x=3.$



8. (5 points) Find the equation of the tangent line to the graph of a function $y=f(x)$ at $x=5$ if $f(5)=-3$ and $f'(5)=4$.

$y = f(5) + f'(5)(x-5)$

$= -3 + 4(x-5)$

$= 4x-23$

9. (5 points) Use the definition of the derivative of a function to show that the derivative of $g(x)=\sqrt{1+2x}$ is $g'(x)=\dfrac{1}{\sqrt{1+2x}}$.

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1+2(x+h)} - \sqrt{1+2x}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1+2x+2h} - \sqrt{1+2x}}{h} \cdot \frac{\sqrt{1+2x+2h} + \sqrt{1+2x}}{\sqrt{1+2x+2h} + \sqrt{1+2x}}$$

$$= \lim_{h\to 0} \frac{(1+2x+2h) - (1+2x)}{h\left(\sqrt{1+2x+2h} + \sqrt{1+2x}\right)}$$

$$= \lim_{h\to 0} \frac{2h}{h\left(\sqrt{1+2x+2h} + \sqrt{1+2x}\right)}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{1+2x+2h} + \sqrt{1+2x}}$$

$$= \frac{2}{\sqrt{1+2x} + \sqrt{1+2x}} = \frac{2}{2\sqrt{1+2x}} = \frac{1}{\sqrt{1+2x}}.$$