① Explain why this function is discontinuous at $x=0$

$$f(x) = \begin{cases} \cos(x) & \text{if } x>0 \\ x & \text{if } x<0 \end{cases}$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \cos(x) = 1$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x = 0$$

Since the two one sided limits are not the same, this limit doesn't exist, so $f(x)$ is discontinuous.

Also, x is not in the domain of the function.

② Find the horizontal and vertical asymptotes of $y = \dfrac{1+x^3}{x^3-2x^2}$

Vertical asymptotes

$$x^3-2x^2=0$$

$$x^2(x-2)=0$$

$$\boxed{x=0,\ x=2}$$

Horizontal asymptotes.

$$\lim_{x\to\infty} \frac{1+x^3}{x^3-2x^2} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{\frac{1}{x^3}+1}{1-\frac{2}{x}} = 1$$

$$\lim_{x\to-\infty} \frac{1+x^3}{x^3-2x^2} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{\frac{1}{x^3}+1}{1-\frac{2}{x}} = 1$$

$\boxed{\text{So horizontal asymptote at } y=1}$