

MATH 141 (Section 3 & 4)
Prof. Meade

University of South Carolina
Fall 2009

Exam 4
November 23, 2009

Name: _____
Section: 003 / 004 (circle one)

Instructions:

1. There are a total of 8 problems (including the Extra Credit problem) on 7 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	14	
2	20	
3	9	
4	8	
5	7	
6	12	
7	15	
8	15	
Total	100	

Happy Thanksgiving!

1. (14 points) Sketch the graph of $f(x) = \frac{x-5}{(x-4)^2}$ on the interval $0 \leq x \leq 10$. Be sure to clearly identify the intervals of increase and decrease, the intervals of concavity, all local maximum and local minimum points, all inflection points, and all asymptotes.

NOTE: $f'(x) = \frac{6-x}{(x-4)^3}$ and $f''(x) = \frac{2(x-7)}{(x-4)^4}$

HINT: $2/9 \approx 0.22$

2. (20 points) Evaluate each of the following limits. Be sure to indicate each time l'Hôpital's Rule is applied.

(a) $\lim_{x \rightarrow 0} \frac{e^{4x} - 1 - 4x}{x^2}$

(b) $\lim_{t \rightarrow 0} \frac{5^t + 3^{-t}}{t^2}$

(c) $\lim_{x \rightarrow \infty} x^2 e^{-x^2}$

(d) $\lim_{x \rightarrow 0^+} (\cos(x))^{1/x^2}$

(e) [Extra Credit (5pts)] $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2}{n} \left(5 - \frac{2i}{n}\right)^3$

HINT: Recognize this limit as a definite integral.

3. (9 points) Find the function f with $f'(t) = 2\cos(t) + \sec^2(t)$ for $-\pi/2 < t < \pi/2$ and $f(\pi/3) = 4$.

4. (8 points) If $\int_1^5 f(x) dx = 12$, $\int_4^5 f(x) dx = 4$ and $\int_2^4 f(x) dx = -2$, find $\int_1^2 f(x) dx$.

5. (7 points) Suppose $g(x) = \int_{-1}^x \frac{u^2 - 1}{u^2 + 1} du$. Find $g'(3)$.

6. (12 points) Consider the region under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$.

(a) Sketch the region.

(b) Estimate the area of this region using three rectangles and right-hand endpoints. Is this estimate smaller or larger than the exact area?

HINT: Draw the three rectangles on the sketch in (a).

(c) Find the exact area of this region.

7. (15 points) Evaluate each indefinite integral.

(a) $\int (2x - e^x) dx$

(b) $\int (y - 1)(2y - 1) dy$

(c) $\int \frac{x}{x^2 + 1} dx$

8. (15 points) Evaluate each definite integral.

(a) $\int_0^1 (\sqrt{t} - t^2) dt$

(b) $\int_0^1 \frac{4}{t^2 + 1} dt$

(c) $\int_{-\pi/2}^{\pi/2} \cos(x)e^{\sin(x)} dx$