MATH 141 (Section 3 & 4)
Prof. Meade

Exam 1
September 9, 2009

University of South Carolina
Fall 2009

Name: Key
SS # (last 4): _____

Instructions:

1. There are a total of 8 problems on 6 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	8	
2	10	
3	16	
4	18	
5	12	
6	12	
7	6	
8	18	
Total	100	

Good Luck!

1. (8 points) Suppose the graph of f is given. Write an equation for each of the graphs that are obtained from the graph of f as follows.

   (a) shift 3 units upward

   $y = f(x) + 3$

   (b) shift 1 unit downward

   $y = f(x) - 1$

   (c) shift 2 units to the right

   $y = f(x-2)$

   (d) reflect about the x-axis

   $y = -f(x)$

2. (10 points) Determine whether the statement is true or false. In either case, give a short explanation why.

   (a) If f is a function, then $f(5x) = 5f(x)$.

   False, for example if $f(x) = x^2$ then $f(5x) = (5x)^2 = 25x^2$
   and $5f(x) = 5(x^2) = 5x^2$

   (b) A horizontal line intersects the graph of a function at most once.

   False, vertical lines can intersect the graph of a function at most once.

   (c) You can always divide by e^x.

   True, because e^x is never zero, in fact, $e^x > 0$.

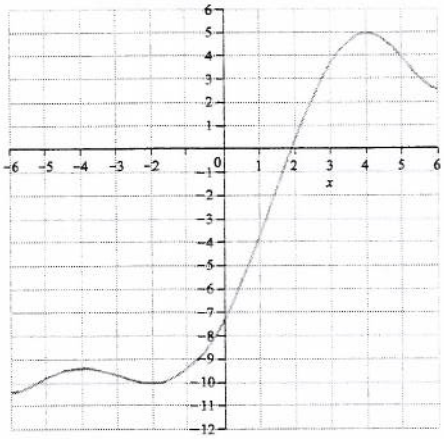
   (d) If $x > 0$, then $\ln(x^6) = 6\ln(x)$.

   True, this is a property of ln (but $x > 0$ is needed!)

   (e) $\tan^{-1}(x) = \dfrac{\sin^{-1}(x)}{\cos^{-1}(x)}$, provided $\cos^{-1}(x) \neq 0$.

   False, this is not how inverse functions work!

3. (16 points) Use the graph of the function f in the figure below to answer the following questions.



(a) Find $f(-2)$. $= -10$

(b) Estimate the value(s) of x such that $f(x) = -4$.

$x = 1$

(c) On what interval(s) is f decreasing?

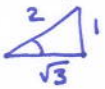
$(-4, -2) \cup (4, 6)$

(d) What is the largest interval containing $x = 0$ on which f is invertible?

$(-2, 4)$

4. (18 points)

(a) Find the exact value of the expression: $\arctan\left(\frac{1}{\sqrt{3}}\right)$. $= \pi/6$

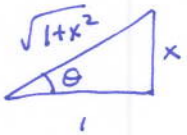


(b) Find the exact value of the expression: $\arcsin\left(\sin\left(\frac{7\pi}{3}\right)\right)$. $= \arcsin\left(\sin\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$

(c) Simplify the expression: $\sin(2\arctan(x))$.

HINT: Draw, and label, a right triangle.

NOTE: Your final answer should be a rational function.



Let $\theta = \arctan(x)$

then $\sin(2\arctan(x)) = \sin(2\theta)$

$= 2\sin\theta\cos\theta$

$= 2\frac{x}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{1+x^2}} = \frac{2x}{1+x^2}$

5. (12 points) If $f(x) = x^2 - 2x + 3$, evaluate the difference quotient

$$\frac{f(a+h) - f(a)}{h}.$$

NOTE: Your final answer should not involve a fraction.

$f(a+h) = (a+h)^2 - 2(a+h) + 3$

$= a^2 + 2ah + h^2 - 2a - 2h + 3$

$f(a) = a^2 - 2a + 3.$

$f(a+h) - f(a) = (a^2 + 2ah + h^2 - 2a - 2h + 3)$

$\quad - (a^2 \qquad - 2a \quad + 3)$

$= 2ah + h^2 - 2h$

$$\frac{f(a+h) - f(a)}{h} = \frac{2ah + h^2 - 2h}{h}$$

$$= \frac{(2a + h - 2)h}{h}$$

$= 2a + h - 2$

6. (12 points) Solve for x: $\ln(x) - \ln(x-1) = 1$.

$\ln(x) - \ln(x-1) = 1$

$\ln\left(\frac{x}{x-1}\right) = 1$

$e^{\ln\left(\frac{x}{x-1}\right)} = e^{1}$

$\frac{x}{x-1} = e$

$x = e(x-1)$

$x = ex - e$

$e = ex - x$

$e = (e-1)x$

$x = \frac{e}{e-1}$

7. (6 points) The table shows the position of a cyclist during the first 10 seconds of a ride.

t (seconds)	0	1	2	4	6	10
s (meters)	0	8.2	17.4	35.2	58.3	95.8

Find the average velocity over the given time periods:

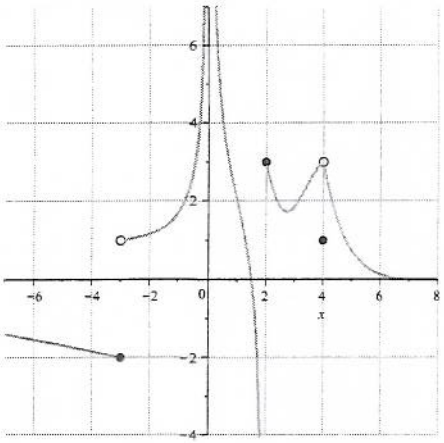
(a) $[1, 6]$

$\frac{\Delta s}{\Delta t} = \frac{s(6) - s(1)}{6-1} = \frac{58.3 - 8.2}{5} = \frac{50.1}{5} = 10.02$ m/sec.

(b) $[0, 10]$

$\frac{\Delta s}{\Delta t} = \frac{s(10) - s(0)}{10-0} = \frac{95.8 - 0}{10-0} = 9.58$ m/sec.

8. (18 points) The graph of f is shown in the figure. Note that f has vertical asymptotes at $x = 0$ and $x = 2$.



Find each number, or explain why it does not exist.

(a) $f(-3)$ = -2

(b) $\lim_{x \to -3^-} f(x)$ -2

(c) $\lim_{x \to -3} f(x)$ dne

(d) $\lim_{x \to 0} f(x)$ = $+\infty$

(e) $f(0)$ dne

(f) $\lim_{x \to 2^+} f(x)$ = 3

(g) $f(4)$ = 1

(h) $\lim_{x \to 4} f(x)$ = 3

(i) $\lim_{x \to \infty} f(x)$ = 0