MATH 141 (Section 11 & 12) Prof. Meade Exam 1 – Practice October 3, 2007 University of South Carolina Fall 2007 Name: ______ Section: 011 / 012 (circle one)

Instructions:

- 1. There are a total of 8 problems on 7 pages. Check that your copy of the exam has all of the problems.
- 2. Calculators may not be used for any portion of this exam.
- 3. You must show all of your work to receive credit for a correct answer.
- 4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	20	
2	14	
3	8	
4	20	
5	6	
6	10	
7	10	
8	12	
Total	100	

1. (20 points) Evaluate the following limits.

(a)
$$\lim_{x \to 0} \frac{\sqrt{x^2 + 4} - 2}{x^2}$$

(b)
$$\lim_{x \to 1} \frac{x^3 - x^2}{x - 1}$$

(c)
$$\lim_{x \to \infty} \frac{2x - 7}{4x - x^2}$$

(d)
$$\lim_{x \to 0} \frac{x \sin(x)}{1 - \cos(x)}$$

- 2. (14 points) Find $\lim_{x \to a} f(x)$ where $f(x) = \sqrt{x-5}$ for
 - (a) a = 0
 - (b) $a = 5^+$
 - (c) $a = -5^{-}$
 - (d) a = 5
 - (e) a = -5
 - (f) $a = \infty$
 - (g) $a = -\infty$
- 3. (8 points) Find all points where $f(x) = \frac{x+3}{|x^2+3x|}$ is not continuous.

4. (20 points) Find the derivative of each of the following functions.

(a)
$$f(x) = x^8 - 3\sqrt{x} + 5x^{-3}$$

(b)
$$f(x) = x^2 \sin(x)$$

(c)
$$f(x) = \frac{\sin(x)}{2x + \cos(x)}$$

5. (6 points) The differentiable functions f and g have f(1) = 1, g(1) = 2, f'(1) = 3, and g'(1) = -1. Evaluate each of the following expressions.

(a)
$$\frac{d}{dx} \left(f(x)g(x) \right) \Big|_{x=1}$$

(b)
$$\left. \frac{d}{dx} \sqrt{f(x)} \right|_{x=1}$$

- 6. (10 points) The amount of water in a tank t minutes after it has started to drain is given by $W = 100(t 15)^2$.
 - (a) What is the average rate at which water flows out of the tank during the first 5 minutes?

(b) What is the instantaneous rate at which water flows out of the tank at the end of 5 minutes?

- 7. (10 points)
 - (a) State the definition of the derivative of a function f(x) in terms of a limit.

(b) Use the definition of the derivative to find dy/dx for $y = \sqrt{9-4x}$.

8. (12 points) The figure below shows the graph of y = f'(x) for an unspecified function f.



(a) For what value of x doe sthe curve y = f(x) have a horizontal tangent line?

(b) Over what intervals does the curve y = f(x) have tangent lines with positive slope?

(c) Over what intervals does the curve y = f(x) have tangent lines with negative slope?