Math 141 (Section 11 \& 12)
Prof. Meade
Exam 1 - Practice
October 3, 2007

University of South Carolina
Fall 2007
Name:
Section: 011 / 012 (circle one)

Instructions:

1. There are a total of 8 problems on 7 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

| Problem | Points | Score |
| :---: | :---: | :---: |
| 1 | 20 |  |
| 2 | 14 |  |
| 3 | 8 |  |
| 4 | 20 |  |
| 5 | 6 |  |
| 6 | 10 |  |
| 7 | 10 |  |
| 8 | 100 |  |
| Total |  |  |

1. (20 points) Evaluate the following limits.
(a) $\lim _{x \rightarrow 0} \frac{\sqrt{x^{2}+4}-2}{x^{2}}$
(b) $\lim _{x \rightarrow 1} \frac{x^{3}-x^{2}}{x-1}$
(c) $\lim _{x \rightarrow \infty} \frac{2 x-7}{4 x-x^{2}}$
(d) $\lim _{x \rightarrow 0} \frac{x \sin (x)}{1-\cos (x)}$
2. (14 points) Find $\lim _{x \rightarrow a} f(x)$ where $f(x)=\sqrt{x-5}$ for
(a) $a=0$
(b) $a=5^{+}$
(c) $a=-5^{-}$
(d) $a=5$
(e) $a=-5$
(f) $a=\infty$
(g) $a=-\infty$
3. (8 points) Find all points where $f(x)=\frac{x+3}{\left|x^{2}+3 x\right|}$ is not continuous.
4. (20 points) Find the derivative of each of the following functions.
(a) $f(x)=x^{8}-3 \sqrt{x}+5 x^{-3}$
(b) $f(x)=x^{2} \sin (x)$
(c) $f(x)=\frac{\sin (x)}{2 x+\cos (x)}$
5. (6 points) The differentiable functions $f$ and $g$ have $f(1)=1, g(1)=2, f^{\prime}(1)=3$, and $g^{\prime}(1)=-1$. Evaluate each of the following expressions.
(a) $\left.\frac{d}{d x}(f(x) g(x))\right|_{x=1}$
(b) $\left.\frac{d}{d x} \sqrt{f(x)}\right|_{x=1}$
6. (10 points) The amount of water in a tank $t$ minutes after it has started to drain is given by $W=100(t-15)^{2}$.
(a) What is the average rate at which water flows out of the tank during the first 5 minutes?
(b) What is the instantaneous rate at which water flows out of the tank at the end of 5 minutes?
7. (10 points)
(a) State the definition of the derivative of a function $f(x)$ in terms of a limit.
(b) Use the definition of the derivative to find $d y / d x$ for $y=\sqrt{9-4 x}$.
8. (12 points) The figure below shows the graph of $y=f^{\prime}(x)$ for an unspecified function $f$.

(a) For what value of $x$ doe sthe curve $y=f(x)$ have a horizontal tangent line?
(b) Over what intervals does the curve $y=f(x)$ have tangent lines with positive slope?
(c) Over what intervals does the curve $y=f(x)$ have tangent lines with negative slope?
