MATH 141 (Section 11 & 12)
Prof. Meade

University of South Carolina
Fall 2007

Exam 1 – Practice
September 13, 2007

Name: Key
SS #:

Instructions:

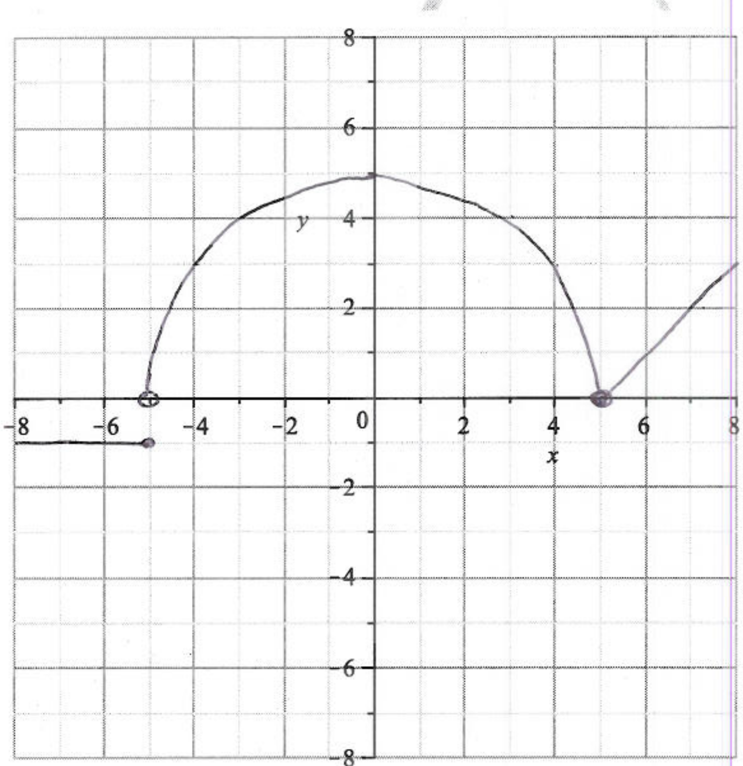
1. There are a total of 8 problems on 6 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	15	
2	16	
3	16	
4	12	
5	9	
6	12	
7	8	
8	12	
Total	100	

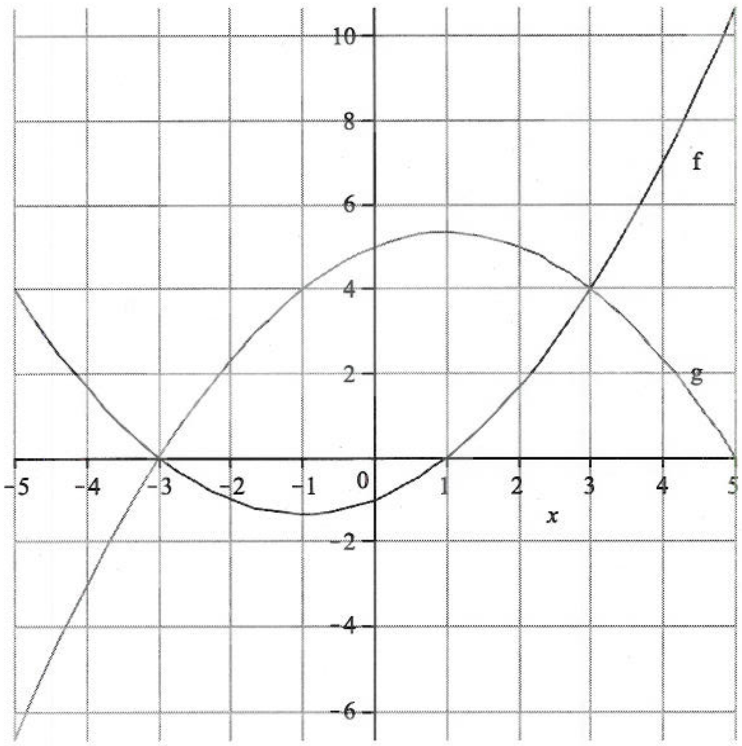
Good Luck!

1. (15 points) Use the axes provided to sketch the graph of the function

$$f(x) = \begin{cases} -1 & x \leq -5 \\ \sqrt{25 - x^2} & -5 < x < 5 \\ x - 5 & x \geq 5 \end{cases}$$



2. (16 points) Use the graphs of the functions f and g in the figure below to answer the following questions.



(a) Find $f(3)$. $=4$

(b) Find $g(-1)$. $=4$

(c) For what values of x is $f(x) = g(x)$? $x = -3$

and $x = 3$

(d) For what values of x is $f(x) < 4$? $-5 < x < 3$

3. (16 points) Let $f(x) = -x^2$ and $g(x) = 1/\sqrt{x}$.

   (a) Find a formula for $f \circ g$.

   $(f \circ g)(x) = f(g(x)) = f(1/\sqrt{x}) = -(1/\sqrt{x})^2 = -\frac{1}{x}$

   defined for $x > 0$

   (b) What is the domain of $f \circ g$?

   all $x > 0$

   (c) Find a formula for $g \circ f$.

   $(g \circ f)(x) = g(f(x)) = \frac{1}{\sqrt{f(x)}} = \frac{1}{\sqrt{-x^2}}$

   defined for all x with $f(x) > 0$

   (d) What is the domain of $g \circ f$?

   since $f(x) = -x^2 \leq 0$

   the domain of $g \circ f$ is empty!

4. (12 points) Determine if each of the following functions is even, odd, or neither.

   (a) $x^2 \sin(x)$ $\quad (-x)^2 \sin(-x) = x^2(-\sin(x)) = -x^2 \sin(x)$

   odd

   (b) $\sin^2(x)$ $\quad (\sin(-x))^2 = (-\sin(x))^2 = (\sin(x))^2$

   even

   (c) $x + x^2$ $\quad (-x) + (-x)^2 = -x + x^2$

   neither

   (d) $\sin(x)\tan(x)$ $= \sin(x) \frac{\sin(x)}{\cos(x)} = \frac{(\sin(x))^2}{\cos(x)}$

   $\frac{(\sin(-x))^2}{\cos(-x)} = \frac{(-\sin(x))^2}{\cos(x)} = \frac{(\sin(x))^2}{\cos(x)} = \sin(x)\tan(x)$

   even

5. (9 points) Let $f(x) = \sin\left(\frac{1-2x}{x}\right)$ for $\frac{2}{4+\pi} \le x \le \frac{2}{4-\pi}$. Find a formula for $f^{-1}(x)$, or explain why the inverse does not exist.

$y = \sin\left(\frac{1-2x}{x}\right)$

$\arcsin(y) = \frac{1-2x}{x} = \frac{1}{x} - 2$

$2 + \arcsin(y) = \frac{1}{x}$

$x = \frac{1}{2+\arcsin(y)}$

$y = \frac{1}{2+\arcsin(x)}$

The inverse function is

$f^{-1}(x) = \frac{1}{2+\arcsin(x)}$

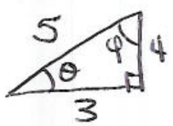
6. (12 points)

(a) Find the exact numerical value for

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

Let $\theta = \arcsin\left(\frac{4}{5}\right)$ & $\varphi = \arccos\left(\frac{4}{5}\right)$.

$$\sin\left(\arcsin\left(\frac{4}{5}\right) + \arccos\left(\frac{4}{5}\right)\right) = \sin\left(\arcsin\left(\tfrac{4}{5}\right)\right)\cos\left(\arccos\left(\tfrac{4}{5}\right)\right) + \cos\left(\arcsin\left(\tfrac{4}{5}\right)\right)\sin\left(\arccos\left(\tfrac{4}{5}\right)\right)$$



$= \frac{4}{5}\cdot\frac{4}{5} + \frac{3}{5}\cdot\frac{3}{5}$

$= \frac{16}{25} + \frac{9}{25} = 1.$

Note that $\theta + \varphi = \pi/2$ so you could also solve this by $\sin(\theta+\varphi) = \sin(\pi/2) = 1$.

(b) Express the following function as a rational function of x:

$$f(x) = 3\ln\left(e^{2x}(e^x)^3\right) + 2e^{\ln(x)}.$$

$= 3\ln\left(e^{2x}e^{3x}\right) + 2e^{\ln x}$

$= 3\ln\left(e^{5x}\right) + 2e^{\ln x}$

$= 3\ln\left((e^x)^5\right) + 2e^{\ln x}$

$= 3\cdot 5\ln(e^x) + 2e^{\ln x} = 15x + 2x = 17x.$

(c) Consider the parametric curve given by $x = 3 + 2\cos(t)$, $y = 2 + 4\sin(t)$ $(0 \le t \le 2\pi)$. Eliminate the parameter t and find y as a function of x.

HINT: Simplify your answer until it has no trigonometric functions.

$x = 3 + 2\cos(t)$

$x - 3 = 2\cos(t)$

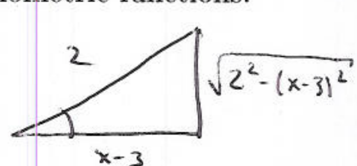
$\frac{x-3}{2} = \cos(t)$

$t = \arccos\left(\frac{x-3}{2}\right)$

$y = 2 + 4\sin(t) = 2 + 4\sin\left(\arccos\left(\frac{x-3}{2}\right)\right)$

$= 2 + 4\frac{\sqrt{2^2 - (x-3)^2}}{2}$

$= 2 + 2\sqrt{4 - (x-3)^2}$



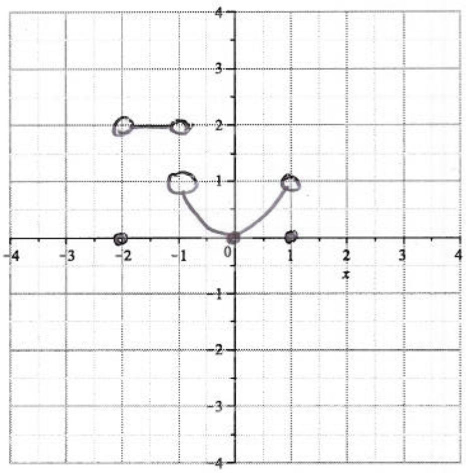
7. (8 points) Sketch the graph of a function f with all of the following properties:

(a) the domain of f is $[-2, 1]$

(b) $f(-2) = f(0) = f(1) = 0$

(c) $\lim_{x\to -2^+} f(x) = 2$, $\lim_{x\to 0} f(x) = 0$, and $\lim_{x\to 1^-} f(x) = 1$.

(d) $\lim_{x\to -1}$ does not exist



8. (12 points) For the function F graphed below, find

(a) $\lim_{x\to 2} F(x)$ dne

(b) $\lim_{x\to 2^-} F(x) = 0$

(c) $\lim_{x\to 2^+} F(x) = -\infty$

(d) $F(2) = 3$