Exam 3 - Practice
October 29, 2004

Name:
Section: 001002 (circle one)

Instructions:

1. There are a total of 6 problems on 6 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

| Problem | Points | Score |
| :---: | :---: | :---: |
| 1 | 25 |  |
| 2 | 25 |  |
| 3 | 15 |  |
| 4 | 15 |  |
| 5 | 10 |  |
| 6 | 10 |  |
| Total | 100 |  |

## Study Hard! Ask Questions!!

1. (25 points) Simplify each of the following expressions.
(a) Solve for $x: \log _{10}\left(x^{2}\right)+\log _{10}(x)=30$.
(b) Rewrite $\ln \sqrt{\frac{x^{2}+1}{x^{3}+5}}$ in terms of simpler logarithms.
(c) Solve for $x: 2 e^{3 x}=7$
(d) Find the exact value of $\tan ^{-1}(1)$.
(e) Use the "triangle method" to find an identity for $\cos \left(\sin ^{-1}(x)\right.$.

2. (25 points) Find the derivative of each of the following functions.
(a) $f(x)=\sin \left(e^{x}\right)$
(b) $f(x)=2^{\left(x^{2}\right)}$
(c) $f(x)=\ln (\ln (x))$
(d) $f(x)=\sin ^{-1}(x)+\cos ^{-1}(x)$
(e) $f(x)=\tan ^{-1}\left(\frac{1}{x}\right)$

3. (15 points) Let $f(x)=(x+1)^{4}$ for $x \geq 0$.
(a) [6 points] Find a formula for $f^{-1}(x)$.
(b) [3 points $]$ State the domain of $f^{-1}$.
(c) [6 points] Sketch graphs of $y=f(x)$ and $y=f^{-1}(x)$ on the axes provided below.

4. (15 points) Evaluate the following limits. Identify each time l'Hôpital's Rule is applied, including the type of indeterminate form.
(a) $\lim _{x \rightarrow 0} \frac{e^{x}-1}{\sin x}$
(b) $\lim _{x \rightarrow 3} \frac{x-3}{3 x^{2}-13 x+12}$
5. (10 points) A function $f$ that is continuous for all real numbers has the following sign chart for its first and second derivatives.

| interval | sign of $f^{\prime}(x)$ | sign of $f^{\prime \prime}(x)$ |
| :--- | :---: | :---: |
| $x<1$ | + | + |
| $1<x<3$ | + | - |
| $3<x$ | + | + |

(a) On what intervals is $f$ increasing?
(b) On what intervals is $f$ decreasing?
(c) On what intervals is $f$ concave up?
(d) On what intervals is $f$ concave down?
(e) Find the $x$-coordinate of all inflection points.
6. (10 points) A stone dropped into a still pond sends out a circular ripple with radius that increases at a constant $3 \mathrm{ft} / \mathrm{s}$. How rapidly is the area enclosed by the ripple increasing after 10 s ?

