MATH 141 (Section 1 & 2) Prof. Meade

Exam 3 – Practice October 29, 2004 University of South Carolina Fall 2004

Name: ______ Section: 001 002 (circle one)

Instructions:

- 1. There are a total of 6 problems on 6 pages. Check that your copy of the exam has all of the problems.
- 2. Calculators may not be used for any portion of this exam.
- 3. You must show all of your work to receive credit for a correct answer.
- 4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	25	
2	25	
3	15	
4	15	
5	10	
6	10	
Total	100	

Study Hard! Ask Questions!!

- 1. (25 points) Simplify each of the following expressions.
 - (a) Solve for x: $\log_{10}(x^2) + \log_{10}(x) = 30$.

(b) Rewrite $\ln \sqrt{\frac{x^2+1}{x^3+5}}$ in terms of simpler logarithms.

(c) Solve for x: $2e^{3x} = 7$

(d) Find the exact value of $\tan^{-1}(1)$.

(e) Use the "triangle method" to find an identity for $\cos(\sin^{-1}(x))$.

2. (25 points) Find the derivative of each of the following functions.

(a) $f(x) = \sin(e^x)$

(b)
$$f(x) = 2^{(x^2)}$$

(c) $f(x) = \ln(\ln(x))$

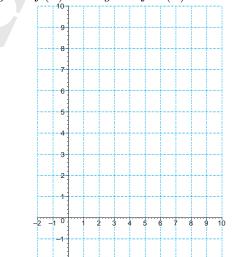
(d)
$$f(x) = \sin^{-1}(x) + \cos^{-1}(x)$$

(e)
$$f(x) = \tan^{-1}\left(\frac{1}{x}\right)$$

- 3. (15 points) Let $f(x) = (x+1)^4$ for $x \ge 0$.
 - (a) [6 points] Find a formula for $f^{-1}(x)$.

(b) [3 points] State the domain of f^{-1} .

(c) [6 points] Sketch graphs of y = f(x) and $y = f^{-1}(x)$ on the axes provided below.



4. (15 points) Evaluate the following limits. Identify each time l'Hôpital's Rule is applied, including the type of indeterminate form.

(a)
$$\lim_{x \to 0} \frac{e^x - 1}{\sin x}$$

(b)
$$\lim_{x \to 3} \frac{x-3}{3x^2 - 13x + 12}$$

(c) $\lim_{x \to \infty} (e^x + x)^{1/x}$

5. (10 points) A function f that is continuous for all real numbers has the following sign chart for its first and second derivatives.

interval	sign of $f'(x)$	sign of $f''(x)$
x < 1	+	+
1 < x < 3	+	-
3 < x	+	+

- (a) On what intervals is f increasing?
- (b) On what intervals is f decreasing?
- (c) On what intervals is f concave up?
- (d) On what intervals is f concave down?
- (e) Find the *x*-coordinate of all inflection points.
- 6. (10 points) A stone dropped into a still pond sends out a circular ripple with radius that increases at a constant 3 ft/s. How rapidly is the area enclosed by the ripple increasing after 10 s?

