>

INTRODUCTION TO MAPLE:
Basic computations, plotting, and differential equations

Doug Meade
meade@math.scarolina.edu (or, just meade on ENGR)
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WHAT IS MAPLE?

Much of your mathematical background has probably be focused on developing the ability to solve equations and explore functions. The sophisticated manipulation of symbols and expressions that you use to solve equations and investigate functions can also be performed by software packages called computer algebra systems (CAS). A CAS can be use to generate many of the exact symbolic solutions you obtained by hand, the numerical approximations you found using a calculator, and the graphs you have drawn. Maple is one of several CASs; other major examples are Mathematica, Macsyma, Derive, and MathCad.

HOW IS MAPLE USED IN MY SECTION OF Math 242?

The first, and largest, use of Maple will be the graphical display of functions, including the solution of one or more differential equations. Two other uses will be as a symbolic calculator and as a programming environment for numerical methods for solving differential equations.

Much of the first week is spent introducing Maple and learning how it can be used as a mathematics tool. I expect each student to be able to use Maple effectively. We will (hopefully) use Maple on a daily basis in this course. You will be encouraged to use Maple on all homework, projects, and exams.

Remember the analogy that a CAS is to calculus as a calculator is to arithmetic.

Note: In previous semesters, students have reported that Maple has been useful in other engineering and science courses!

DOCUMENTATION and ON-LINE HELP

The on-line help for Maple is very good -- once you learn how to use it. The help pages describe the syntax of each command, a brief description of the algorithm that has been implemented, and a few examples illustrating the use of the command. The examples can be very helpful. I will often cut and paste an example from a help page into my worksheet, confirm that the example works as advertised, then modify the arguments to answer my question.

To obtain help on the command func, type: ?func.

Of course, if you don’t know the name of the command, a different approach must be used. I find

the Keyword Search (located under the Help menu) particularly helpful.

The set of Maple references is rapidly growing. I have a number of these books. Of the ones I own, my favorites are
  Introduction to Maple, by Heck (Springer-Verlag, 1993)
  The Maple Handbook, by Redfern (Springer-Verlag, 1993)
  Theoretical Methods in the Physical Sciences, by Baylis (Birkhauser, 1994)

WHERE CAN I USE MAPLE?

Maple is available on many PCs, Macs, and workstations across the USC campus, including SWGN 1D11, 1D15, 1D29, 300Main B203, PSC 106, LC 310, 311, 412. For more specific information, please consult the local support staff.

The Student Edition of Maple may be purchased from the Russell House Bookstore for approximately $90. If you don’t see Maple on the shelves, additional copies can be ordered from the Book Information desk. (The full Maple costs more than $400. The difference is only in the size of the objects that can be manipulated. The limits in the Student Edition are more than enough for our needs!)

USC does have a site license for Maple. Under this agreement any faculty or staff may obtain a free copy of the full (profesional) version of Maple from CSD (contact Alma Kinzley or Barbara Koski at 7-6015).

User support is to be provided through CSD. Another forum for the discussion of Maple is the Maple User’s Group, an Internet-based mailing list. To subscribe, send a message to majordomo@daisy.uwaterloo.ca, with the message body containing the line:
        subscribe maple-list [<address>]

GETTING STARTED

To run Maple, open the Maple V, Release 3 folder then double-click on the icon labelled Maple V R 3 for Windows. (You will almost always want to Maximize the window.)

To open a workseet, select Open from the File menu, then locate the file you want to load. (You can also click on the icon that looks like an opening file folder located along the top of the Maple window. The icon for New is the blank page at the far left.)

Once in a worksheet the current location is marked by a vertical bar. You can move around by moving the mouse and then clicking the left button once, by using the arrow keys, or by using the up and down arrows on the scroll bar located along the right side of this window followed by finer adjustment with the mouse. A command is entered and executed by positioning the cursor anywhere on the desired line and pressing the return key.

It is a good idea to begin EVERY worksheet with the following two commands. Information about these commands can be found in the on-line help: ?restart, ?plots, ?DEtools. For a complete list of available packages, see ?index,package and ?share

```
> restart:
> with( DEtools ): with(plots):
>
```

BASIC FACTS

Maple is a computer language; it cannot read your mind. Maple does follow your instructions to the letter. This means that you need to learn to communicate your needs to Maple in a way that it understands.

Some basic symbols and other fundamentals:

- assignments are made with := (plain = has a different meaning)
  -- this gives the value of the right hand side (RHS) to the name on the LHS

- every command is terminated by a semi-colon (;) or colon (:)
  -- with the latter the computation is done, but the result is not displayed

- the double quote (") refers to the result of the immediately preceding computation

- Maple is case sensitive
  -- that is, the names x and X are different, pi and Pi are not the same thing

- { }
  -- set notation

- .. (as in a .. b)
  -- this is how Maple indicates the interval [a, b], that is, the real numbers from a to b

- ? or help
  -- a request to Maple for information

The standard mathematical functions are denoted by their standard names:
+ (plus) , - (minus) , * (times) , / (divided by) , ^ (raised to the power) , sin, cos, tan, abs, sqrt

Common constants are predefined in Maple, their names are: Pi, E, and I
(Yes, they have to be capitalized exactly like this; PI, pi, e, and i do not refer to these constants.)

Unless specified otherwise (with assume), all names are assumed to be complex valued. This is not often a problem, but there are times when this can lead to surprising results. (We'll see an example of this during this demonstration.)

In the following lines, use paper and pencil to first predict what you think Maple will do. Then execute the command and see what actually happens!

```
>
> number := 4 *  6 + 12 /  6 - 1 ;
> power := (-3)^3 ;
> abs( " );
> Pi ;
> v:= sin( Pi / 4 ) :
> w:= v^2  ;
> v;
> tan(  -Pi / 2 ) ;
> 3 / ( 5 - sqrt( number ) ) ;
>
```

Now enter some commands of your own! To produce the input prompt either select Insert New Region (Below) from the Format menu, click on the eighth icon from the left, or simply press CTRL-I. Any of these will produce a new input region below the current cursor position. Be generous with the space bar to make your commands easy to read, and to edit (modify after the fact). Go back to the second tangent calculation above (remember how to reposition the cursor!) and change it to compute the tangent of Pi / 3.

Now for some slightly more impressive computations

```
> 2^100;
> evalf( Pi, 250 );
> 400!;
>
```

Now let's define a few functions and see how Maple operates with these. (The only difference is that in these examples the name x does not have a value.)

```
>
> f := sin( 2  * x ) ;
> g := 4 * x^2 ;
> h := 0.25 * cos ( 8 * x ) ;
> p := f * g ;
> u1 :=  f * h ;
> u2 := f + h ;
>
```

PLOTTING

Graphs of functions are produced by the plot command. In its simplest form, plot needs to know the function to be plotted and the range of values for the independent variable. Note that a..b is Maple's way of describing the interval [a,b]. Observe that I have placed signed titles on some of these graphs. When you actually print plots to turn in with your homework, they MUST have a title and signature [by your initials or last name]. Again, try to predict the output before tapping the return key!

```
>
> plot( 3 * t - 2 , t = -3 .. 10, title='My First Maple Plot' ) ;
> plot( f , x = -Pi .. 2 * Pi ) ;
> plot( p , x = -6 .. 8 , title = ' f * g [by DM] ' );
>
```

These plots are nice, but what information do they convey? Let's concentrate on the plot of p, that is f*g. Position the cursor on a point on the graph and click the left button. The numbers that appear on the lower-left corner are the coordinates of the current location of the cursor. Use this technique to identify the global maximum and minimum values of f*g on the interval [-6, 8], and the x-values at which these are found. Where do other (local) minima and maxima occur? Can you guess the exact values?

```
>
> plot(2 *  u^3 + 4 * u - 5 , u = -10  .. 7 , title = ' cubic function [by DM] ' ) ;
>
```

Where does the cubic function cross the u-axis? Maybe it would be helpful to cut down the plotting interval from [-10, 7] to [-1, 3], or even narrower, say to [0, 1.5]. Try it. Can you find the u-intercept to 2-decimal point accuracy by this process? It is a powerful method, which we call ZOOMING IN.

```
>
> plot( u1 ,  x = -1.2 .. 1.2 );
> plot( u2 , x = 0 .. 3*Pi ) ;
>
```

Can you explain why the graphs of u1 and u2 look the way they do?

You probably now have a big pile of plots cluttering up your screen. Ones that you don't need can be closed temporarily by minimizing the window. To permanently get rid of plots that you no longer need, position the cursor inside the plot window, select Exit from the File menu on the plot window or use the key sequence (ALT-f, x).

PLOTTING MULTIPLE FUNCTIONS IN ONE PLOT

To plot more than one function in a single window, just specify the functions to be plotted as a set.

```
> plot( { p , g , -g } ,  x= -6 ..  8 );
>
```

Can you identify the three different plots? What about the next one? Can you explain the little bumps that appear inside the large dips in the graph of s?

```
> plot( { f , h , u2 }, x=0 .. 3*Pi );
>
```


DISCONTINUOUS FUNCTIONS

The tangent function is well-known to all of us.
Note: This definition completely replaces the previous definition of f.

```
> f :=  tan(x);
>
```

Recall that the graph of y=tan(x) has vertical asymptotes at all odd multiples of Pi/2. Let's see how Maple handles this.

```
> plot( f , x = -Pi .. Pi );
>
```

Is this what you expected to see? Why does the graph look like this? What can be done to improve the appearance of the plot?

There are two problems. First, we have to tell Maple that this function is discontinuous. But, there is more. Look at the labels on the vertical axis. The units are in BILLIONS; we want to see the details on a much finer scale. Let's limit the vertical scale to the range from -10 to 10. Each of these pieces of information is an optional argument to plot.

```
> plot( f,  x = -Pi .. Pi ,  -10 .. 10 , discont=true );
>
```

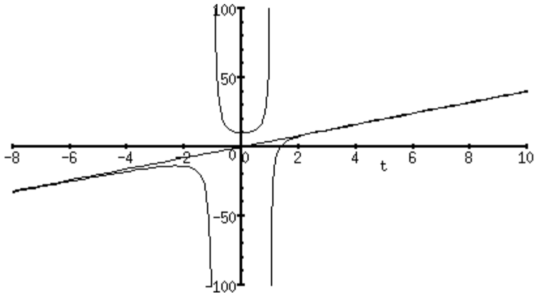
This technique is very useful for any function that has vertical asymptotes.

OK. Now it's your turn. Add appropriate optional arguments to the following command to produce a graph from which you can identify the local minimum and local maximum. (Don't forget the title.)

```
> plot ( ( 10 -  4*t^3) / ( 1 - t^2 ) , t = -3 .. 5 );
>
```

Now, one last challenge. Use your graph to estimate the (oblique) asymptote for this function. Check your estimate by graphing the function and the asymptote in the same plot. (Here's an example of what I'm after, and to show how plots look when inserted into a worksheet.)

```
>
```



These examples only scratch the surface of Maple's abilities when it comes to plotting. Maple can also produce 3-D plots (see ?plot3d), plots of parametric curves and surfaces, animated sequences (see ?animate or ?display or ?animate3d or ?display3d). The best general source of information is the on-line help for the plots package.

```
> ?plots
```

diff, int, limit and all that

```
> restart;
> x := exp(-alpha*t)*cos(omega*t+theta);
> dx := diff( x, t );
> dx2 := diff( x, t$2 );
> factor( dx );
> CP := solve( dx=0, t );
> subs( t=CP, dx2 );
> simplify( ", trig );
```

Since this will be negative (for real values of omega, alpha, and theta), this critical point must be a (local) maximum. The value of the function at this maximum is

```
> X[max] := simplify( subs( t=CP, x ), trig );
> Int( x, t );
> value( "  );
```

Definite integrals are very similar. Here's an example taken from the CRC Tables of Integrals.

```
> Int( u/sin(u), u=0..Pi/2 ):
> " = value( " );
```

The presence of complex numbers in this result is (at first) surprising. It is not difficult to see that this quantity is real.

```
> evalc( " );
```

Now, what is Catalan? (see ?Catalan)

This example illustrates a very important feature of Maple. Even though problems are stated in terms of real quantities, the computational algorithms sometimes require the use of complex analysis. It is also very important to remember that all names are, by default, assumed to be complex valued. Many common calculations actually require an additional assumption on a parameter.This can be very frustrating. (It is one of the most common questions that I see on the Maple User's Group.)

```
> int( exp(-u^2), u=0..infinity );
> int( exp(-a*u^2), u=0..infinity );
```

Note that Maple is unable to evaluate this limit until something is known about the parameter a.

```
> assume( a>0 );
  about( a );
> int( exp(-a*u^2), u=0..infinity );
```

The following assignment is used to reset the name a to its unassigned state.

```
> a := 'a';
  about( a );
> L := Limit( tan(theta), theta=Pi/2 );
> value( L );
> limit( tan(theta), theta=Pi/2, right );
```


An Application: INTEGRATION OF A RATIONAL FRACTION

```
> restart;
> F := (x^2-26*x-47)/(x^5+5*x^3+3*x^4+11*x^2-20);
```

Let's investigate several different approaches to the evaluation of the following integral

```
> Int( F, x );
```

The simplest, but least instructive, approach is to simply let Maple do all the work.

Note that the constant of integration is not included in indefinite integrals.

```
> value( " );
```

or, even,

```
> int( F, x );
```

To emphasize integration techniques while still avoiding algebraic complications, we can utilize Maple to determine the partial fraction expansion of the integrand.

```
> Fpf := convert( F, parfrac, x );
```

From this partial fraction expansion it is now a relatively simple matter to compute the integral BY HAND. The result can be compared with the previous answer

```
> int( Fpf, x );
```

The final approach to this problem will be the most detailed. The purpose here is to emphasize the process of finding a partial fraction expansion without fear of unmanageable algebra.

The key here is to enter the correct form for the partial fraction expansion

```
> factor( denom( F ) );
> FORM := a/(x-1) + b/(x+2) + c/(x+2)^2 + (d+e*x)/(x^2+5);
> eqn := F = FORM;
> simplify( " );
> collect( ", x );
> eq1 := a+b+e=0;
  eq2 := 4*a+b+c+d+3*e;
  eq3 := 9*a+3*b-c+3*d = 1;
  eq4 := -4*e+5*b+5*c+20*a=-26;
  eq5 := -5*c-10*b-4*d+20*a = -47;
> COEFFS := solve( {eq1,eq2,eq3,eq4,eq5}, {a,b,c,d,e} );
> subs( COEFFS, FORM );
```

Let's confirm the answer

```
> simplify( " );
```

The integral is now much simpler to evaluate by hand, but here it is one last time according to Maple.

```
> int( ", x );
>
```

Note that this same outline can be used for ANY partial fraction problem. The key steps are i) knowing the correct form for the expansion and ii) understanding how to identify the conditions that the coefficients must satisfy. These are (relatively) high level concepts; the user is relieved of the rather complicated mechanic manipulations.

DIFFERENTIAL EQUATIONS

The built-in differential equation solver is dsolve. I, however, wish to first emphasize some of the graphical tools for the analysis of ODEs.

```
> restart;
  with( DEtools );
> ODE := diff( y(t), t ) = sin(t) - y(t)/2;
```

Let's begin with the direction field

```
> DEplot( ODE, [t,y], t=0..12, y=-3..3, arrows=THIN, title = ' y'=sin(t)-y/2 ' );
```

To add a few trajectories to the figure it is necessary to provide a set of initial conditions

```
> IC := { seq( [0,i], i=-3..3 ) };
```

NOTE: The order of elements in a set is unpredictable.

```
> DEplot( ODE, [t,y], t=0..12, IC, arrows=THIN, title = ' y'=sin(t)-y/2 ' );
```

A nice exercise for students is to encourage them to change different parameters in the problem and describe any changes in the solution.

Additional optional arguments can be used to fine tune the display; see ?DEplot and ?DEtools[options] for more information.

```
> ?DEplot
> ?DEtools[options]
```

And now, the exact solutions, with and without an initial condition

```
> dsolve( ODE, y(t) );
> dsolve( { ODE, y(0)=1/2 }, y(t) );
```

LAPLACE TRANSFORM

In this example I wish to demonstrate both the syntax for requesting that Laplace transforms be used in the solution procedure and some of Maple's abiliities to work with step and impulse functions. For each problem the solution is determined both by Maple's default algorithm and by Laplace transforms. It can be interesting to compare these solutions.

```
> F1 := 2;
> F2 := 2 + Dirac(t-Pi);
> F3 := 2 + 2*sin(t-Pi)*(Heaviside(t-Pi) - Heaviside(t-3*Pi));
> ODE1 := diff(y(t),t$2) + 5*diff(y(t),t) + 6*y(t) = F1:
> ODE2 := diff(y(t),t$2) + 5*diff(y(t),t) + 6*y(t) = F2:
> ODE3 := diff(y(t),t$2) + 5*diff(y(t),t) + 6*y(t) = F3:
> IC := y(0)=1, D(y)(0)=2:
>
> y1 := rhs( dsolve( { ODE1, IC }, y(t) ) );
> y1L := rhs( dsolve( { ODE1, IC }, y(t), method=laplace) );
> y2 := rhs( dsolve( { ODE2, IC }, y(t) ) );
> y2L := rhs( dsolve( { ODE2, IC }, y(t), method=laplace) );
> y3 := rhs( dsolve( { ODE3, IC }, y(t) ) );
> y3L := rhs( dsolve( { ODE3, IC }, y(t), method=laplace) );
> plot( { F1, y1, y1L }, t=0..3*Pi, y=0..4 );
> plot( { F2, y2, y2L }, t=0..5*Pi, y=0..4 );
> plot( { F3, y3, y3L }, t=0..15*Pi );
```


SYSTEMS OF ODES

This example is taken from a worksheet developed by Matt Miller for Math 141. All references are to the Calculus in Context text.

We use Maple's DEtools to study solutions of the Lotka-Volterra system of problem 6 on page 190. Here h represents the hare population and u represents 60 times the lynx population. Do you understand how u and u' replace L and L' in the differential equations? We are going to work with three different initial equations. Modifications of this program can be used to study the May model (page 189) and the fermentation system (page 192).

```
> restart;
  with( DEtools ):
> rate_equations:=  [ diff(h(t),t) = (0.1)*h-(0.005)*h*(1/60)*u,  diff(u(t),t) = (0.00004)*h*u-(0.04)*u
  ] ;
  vars:= [t, h, u];  # another way to write this would be vars:= [h(t) , u(t) ] ;
> init1:= [0, 2000, 600];  init2:= [0, 2000, 1200 ];  init3:= [0, 2000, 3000 ] ;
  domain := 0 .. 320;
```

First we plot the hare and lynx populations jointly against time using the first of the given initial conditions. You should repeat this with the other initial conditions. Feel free to experiment with different time intervals, different step sizes, etc.

```
> L:= DEplot( rate_equations , vars, t= domain , { init1 },  stepsize= 0.5, scene= [t, u] ):
  H:=  DEplot( rate_equations  , vars, t= domain ,  { init1 },  stepsize= 0.5, scene= [t, h] ):
> plot( {L,H} , title = 'Hares and 60 * Lynxes vs. time by [m^2]' );
```

Next we plot the hare and lynx populations against one another in what is called a PHASE PORTRAIT. We do this for several different initial conditions. Can you identify which curve goes with which initial condition? How is the independent variable t showing up in these pictures? (HInt: try it again with time interval t = 0 .. 20 .)

```
> DEplot( rate_equations , vars, t= 0 .. 160 , {init1, init2, init3 }, stepsize= 0.5, scene= [h,u] , titl
  e =  'Hares vs. 60 * Lynxes for t = 0 .. 160 by [m^2]');
```

Three dimensional phase portraits are also possible.

```
> DEplot( rate_equations , vars, t= 0 .. 160 , {init1, init2, init3 }, stepsize= 0.5, scene= [t,h,u] , ti
  tle =  'Hares vs. 60 * Lynxes for t = 0 .. 160 by [m^2]');
```

This concludes my presentation. As there are substantial parts of this worksheet that have not been discussed in this session, you are encouraged to return to the parts that interest you. Don't forget to consult the on-line help.

END OF WORKSHEET

```
>
```