INTRODUCING MAPLE IN THE CLASSROOM (day1.mws)

Douglas Meade (meade@math.sc.edu)
Matthew Miller (miller@math.sc.edu)
Robert Murphy (murphy@math.sc.edu)

>

Introduction to Maple:

>

Objectives:

- Enter mathematical expressions in Maple
- Plot a function in Maple
- Extract information from a graph
- Learn the solve and other commands for algebra and caclulus

>

WHAT IS MAPLE AND HOW CAN IT BE USED?

Much of your mathematical background has probably been focused on solving equations and graphing functions. The manipulation of symbols and expressions that you use to solve equations and investigate functions can also be performed by software packages called computer algebra systems (CAS). A CAS can be used to produce the exact symbolic solutions you obtained by hand, the numerical approximations you found using a calculator, and the graphs you have drawn. Maple is just one CAS; other major examples are Mathematica, Macsyma, Derive, and MathCad. Probably for students, the greatest use of Maple is the graphical display of functions. In fact, this graphical information is often more useful than a formula. You can also use Maple to find roots of equations, optimize functions, estimate derivatives and integrals numerically, compute derivative and integral formulas, plot and analyze data, and visualize geometric objects and transformations.

Sometimes we will use Maple for demonstrations during class; other times you will use the computers yourselves. Maple will often be useful in the homework, and you can expect exam questions that cannot be done without Maple. For the most part, however, the choice whether or not to use Maple will be up to you; one of the goals of this course is to learn how, and even more importantly, when, to use this powerful tool.

>

DOCUMENTATION and ON-LINE HELP

The on-line help for Maple is very good. The help pages describe the syntax of each command, a brief description of the algorithm that has been implemented, and a few examples illustrating the use of the command. To obtain help on the command func, type: ?func. The Help Browser, an interactive interface to the entire Maple help system, can be brought up by clicking on Help, found at the top right of the Maple window.

>

BASIC FACTS

Maple is a computer language; it cannot read your mind. You need to learn how to communicate with Maple.

The basic symbols:

- assignments are made with := (plain = has a different meaning) -- think of this as giving the value of the right hand side to the name that appears on the left hand side
- every command is terminated by a semi-colon (;) or colon (:) -- with the latter the computation is done, but the result is not displayed
- the percent sign (%) refers to the result of the immediately preceding computation
- Maple is case sensitive -- that is, the names x and X are different, pi and Pi are not the same thing
- { } -- set notation (mostly used in the context of plotting a bunch of functions simultaneously)
- .. as in a .. b -- this is how Maple indicates the interval [a, b], that is, the real numbers from a to b
- ? or help -- a request to Maple for information

The standard mathematical functions are denoted by their standard names:
+ (plus) , - (minus) , * (times) , / (divided by) , ^ (raised to the power) , sin, cos, tan, abs, sqrt, exp, ...

>

GETTING STARTED

You have already learned how to open up a worksheet. After Maple is opened, position the cursor arrow on the menu item File and click the left button once. You can select New to obtain a fresh, empty worksheet. Or you can select Open to choose an existing worksheet (the name will always end with `.mws`), either by double clicking on the name or by selecting the name and then Load. Once in a worksheet the current location is marked by a vertical bar. You can move around by moving the mouse and then clicking the left button once, by using the arrow keys, or by using the scroll bar up and down arrows on the right side of this window followed by finer adjustment with the mouse. A command is entered and executed by positioning the cursor anywhere on the desired line and pressing the return key.

>

Basic math

It is probably a good idea to begin EVERY worksheet that you create with the restart command. Information about this command can be found in the on-line help: ?restart

```
> restart:
```

In the following lines, use paper and pencil to first predict what you think Maple will do. Then execute the command and see what actually happens! If you haven't seen it before the ! notation means multiply all the numbers down to 1; for example 4!=(4)(3)(2)(1).

```
> number:= 4 *  6 + 12 /  6 - 1 ;
```

$$number := 25$$

```
> power:= (-3)^3 ;
```

$$power := -27$$

```
> abs( % );
```

$$27$$

```
> Pi ;
```

$$\pi$$

```
> v:= sin( Pi / 4 ) :
> v;
```

$$\frac{1}{2}\sqrt{2}$$

```
> tan(  -Pi / 2 ) ;
Error, (in tan) singularity encountered
> 3 / ( 5 - sqrt( number ) ) ;
Error, division by zero
> 4! ;
```

$$24$$

```
> 50! ;
```

$$30414093201713378043612608166064768844377641568960512000000000000$$

```
> Digits:= 80:  evalf(Pi);
```

$$3.1415926535897932384626433832795028841971693993751058209749445923078164062862090$$

```
> Digits:= 10:  evalf(Pi^2/6);
```

$$1.644934068$$

```
> f:= 2 * x^2 - 7 * x + 5 ;
```

$$f := 2\,x^2 - 7\,x + 5$$

```
> solve( f = 0 , x ) ;
```

$$1, \frac{5}{2}$$

```
> factor( f ) ;
```

$$(x-1)\,(2\,x-5)$$

```
> solve( z = 3 * t + 4 , t );
```

$$\frac{1}{3}z - \frac{4}{3}$$

```
> solve( a * x^2 + b * x + c = 0 , x ) ;
```

$$\frac{1}{2}\,\frac{-b+\sqrt{b^2-4\,a\,c}}{a}, \frac{1}{2}\,\frac{-b-\sqrt{b^2-4\,a\,c}}{a}$$

```
> expand( (a + b) ^ 3 );
```

$$a^3 + 3\,a^2\,b + 3\,a\,b^2 + b^3$$

Now enter some commands of your own! To produce the input prompt > , use the left mouse button to select the menu item Edit, followed by the submenu item Insert Prompt.. This produces a new input region. Be generous with the space bar to make your commands easy to read, and to edit (modify after the fact). Go back to the tangent calculation above and change it to compute the tangent of $\frac{\pi}{3}$.

```
>
```

Plotting

Next we define some functions for later use.

```
> f1 := sin(x) ;   f2 := sin( 2  * x ) ;
```

$$f1 := \sin(x)$$

$$f2 := \sin(2\,x)$$

```
> g1 := 2 * x^2 ;
> g2 := 2 * x^2  + 3 ;
> g3 := 2 * (x + 3 )^2 ;
```

$$g1 := 2\,x^2$$

$$g2 := 2\,x^2 + 3$$

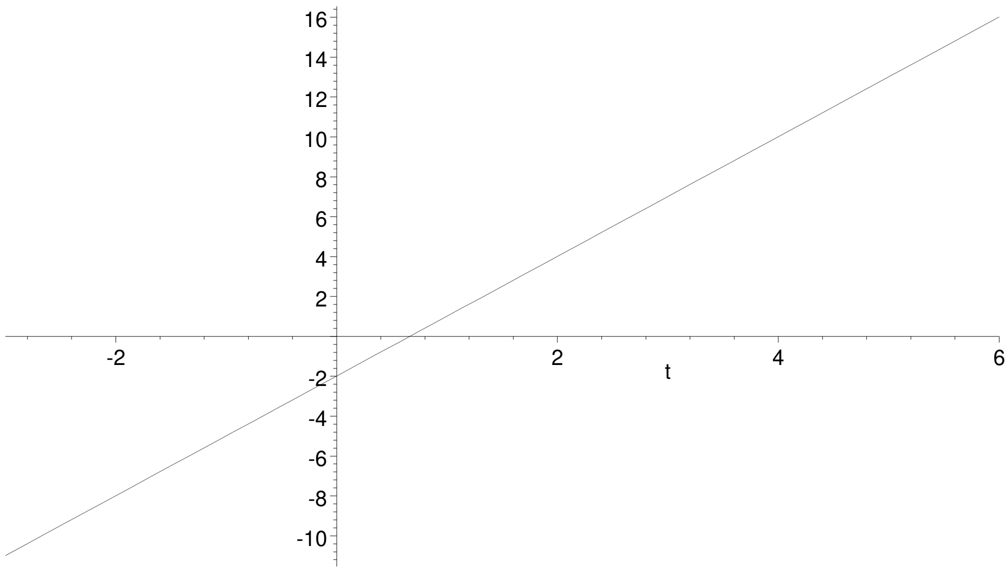
$$g3 := 2\,(x+3)^2$$

```
>
```

Plotting one function

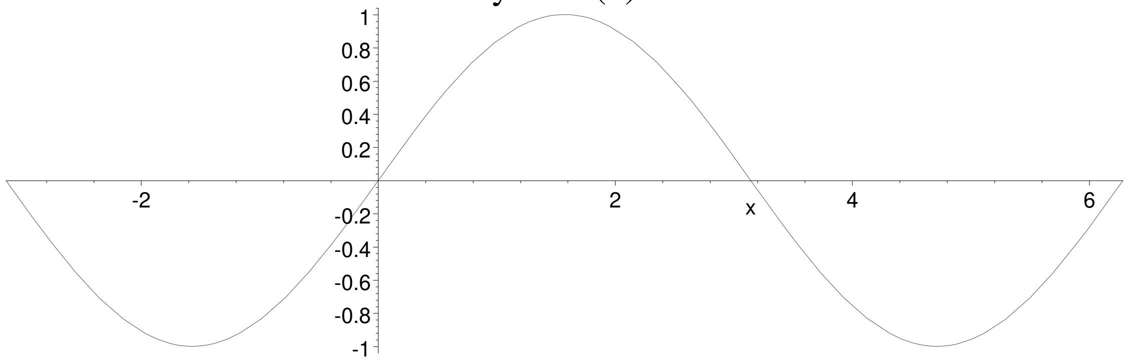
Graphs of functions are produced by the plot command. In its simplest form, plot needs to know the function to be plotted and the range of values for the independent variable. Note that a..b is Maple's way of describing the interval [a,b]. Again, try to predict the output before tapping the return key!

```
> plot( 3 * t - 2 , t = -3 .. 6 ) ;
```



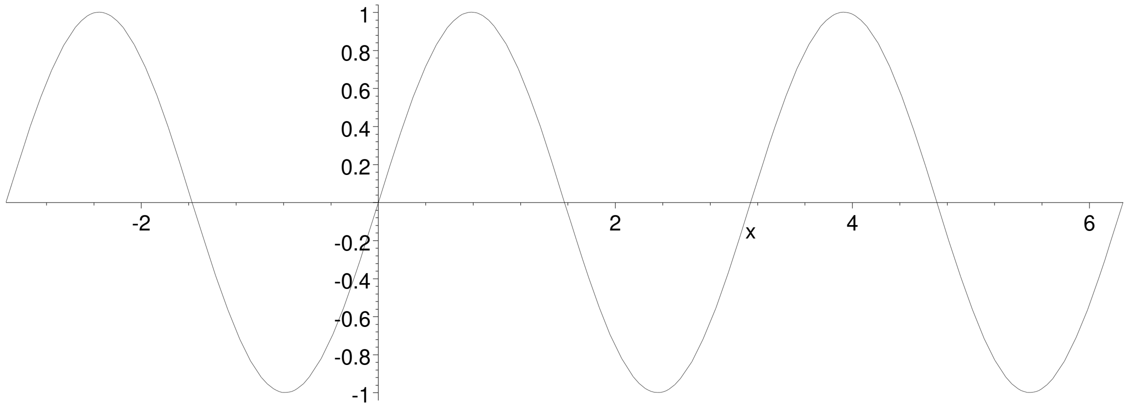
```
> plot( f1 , x = -Pi .. 2 * Pi , title = `y = sin(x)`) ;
```

y = sin(x)



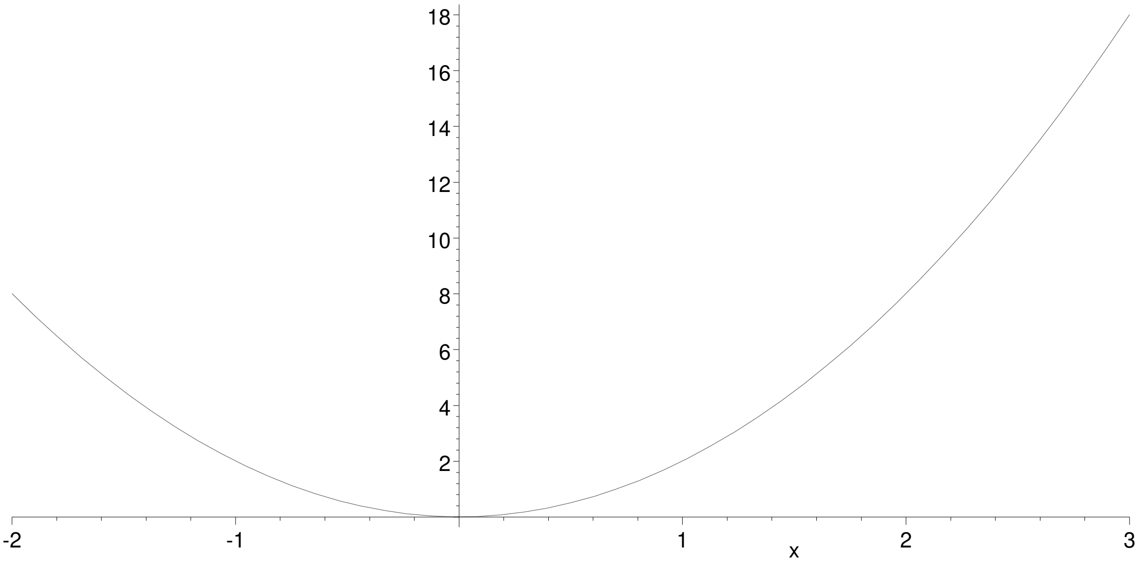

```
> plot( f2 , x = -Pi .. 2 * Pi , title = `y = sin(2x)`);
```

y = sin(2x)



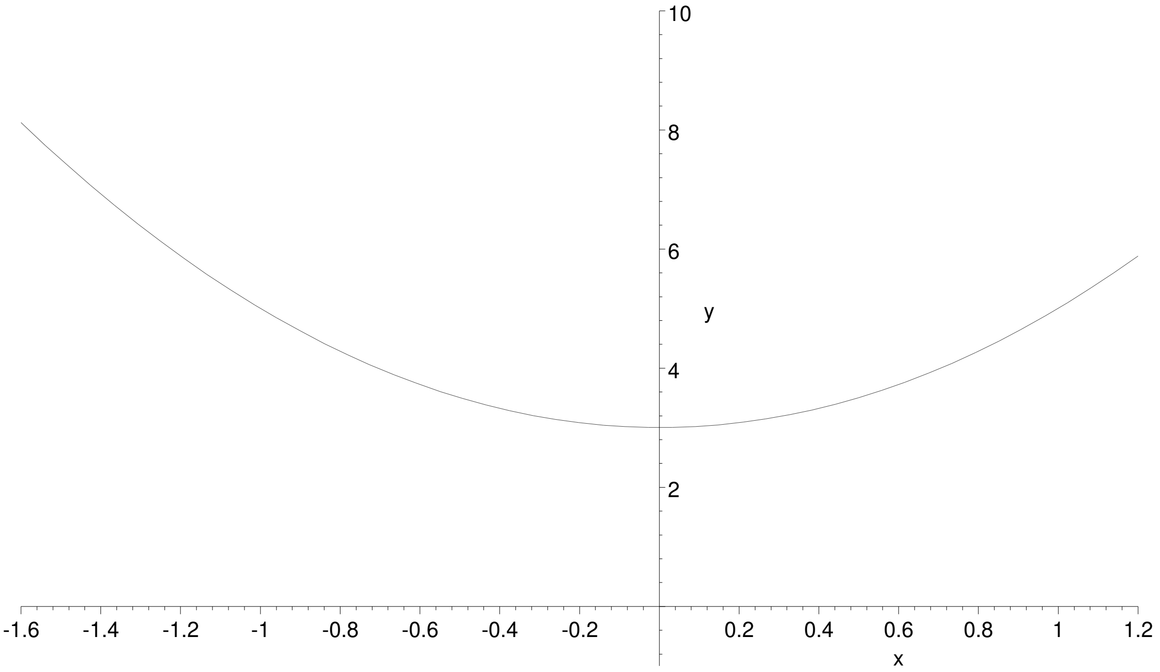

```
> plot( g1 , x = -2 .. 3 , title = `y = 2x^2` );
```

y = 2x^2



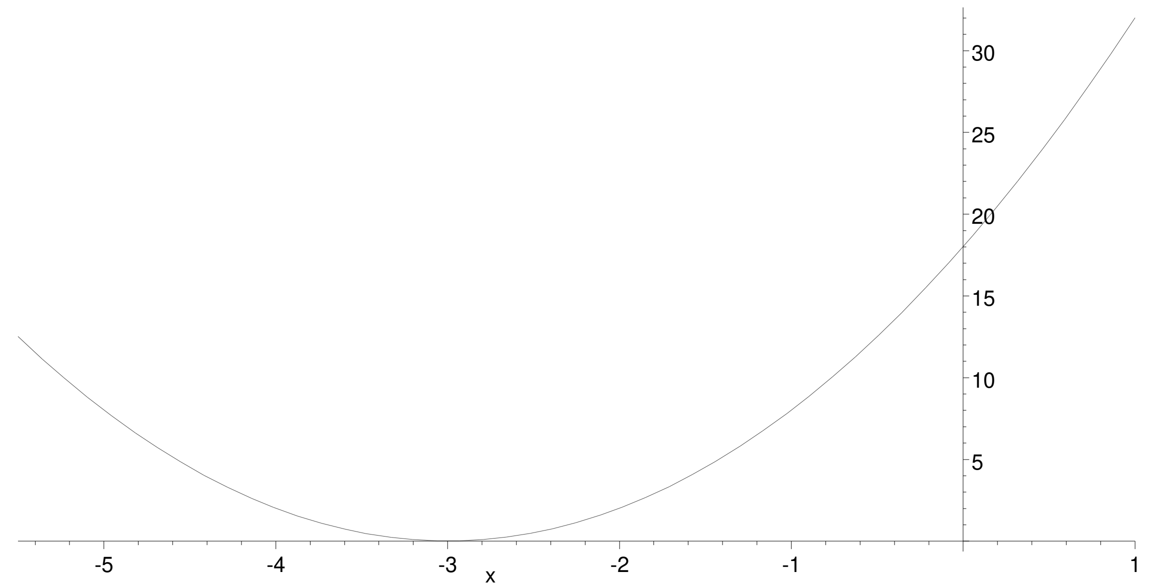
```
> plot( g2 , x = -1.6 .. 1.2 , y = -1 .. 10 , title = 'y = 2x^2 + 3' );
```

y = 2x^2 + 3




```
> plot( g3 , x = -5.5 .. 1, title = 'y = 2( x + 3 )^2'  );
```

y = 2(x + 3)^2

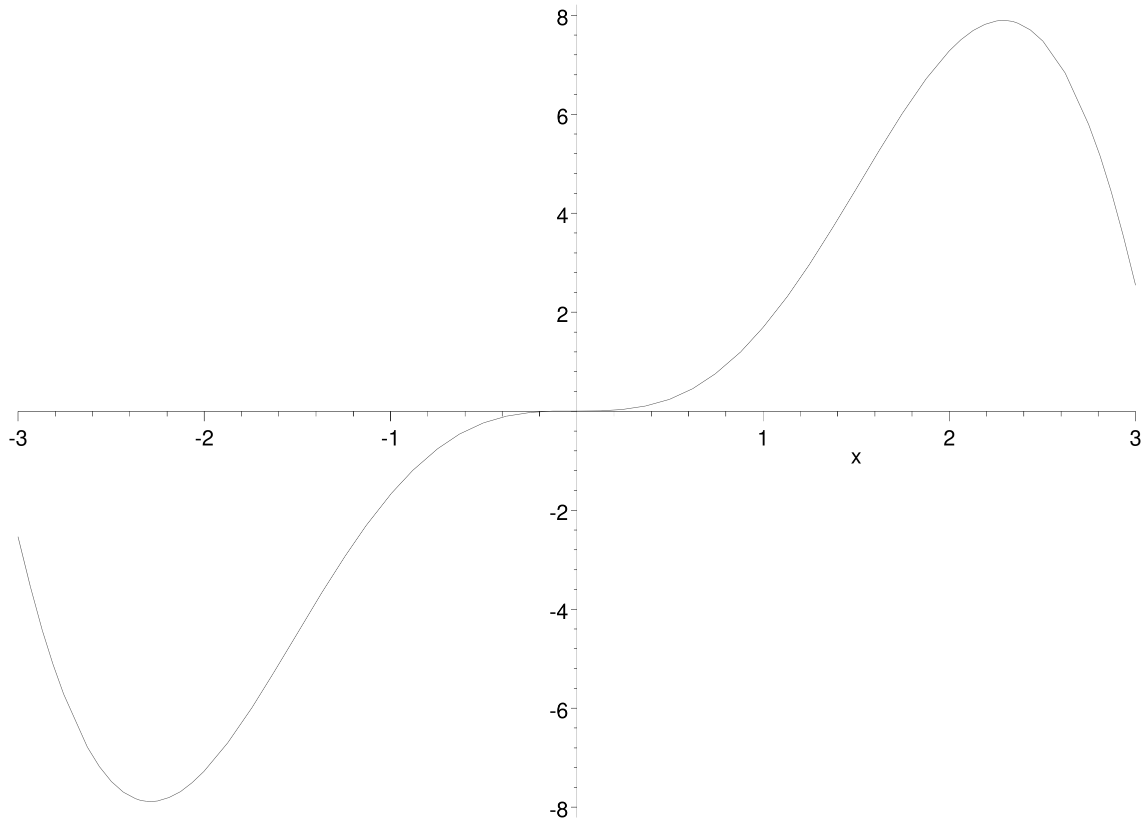


```
> p := f1 * g1 ;
```

$$p := 2\sin(x)\,x^2$$

```
> plot( p , x = -3 .. 3 , title = `p = f1 * g1` );
```

p = f1 * g1



These plots are nice, but what information do they convey? Let's concentrate on the plot of p. Position the cursor on a point on the graph and click the left button. The numbers that appear on the upper-left corner are the coordinates of the current location of the cursor. Use this technique to identify the maximum and minimum values of p(x) on the interval [-3, 3], and the x-values at which these are found.

Plotting multiple functions in one plot

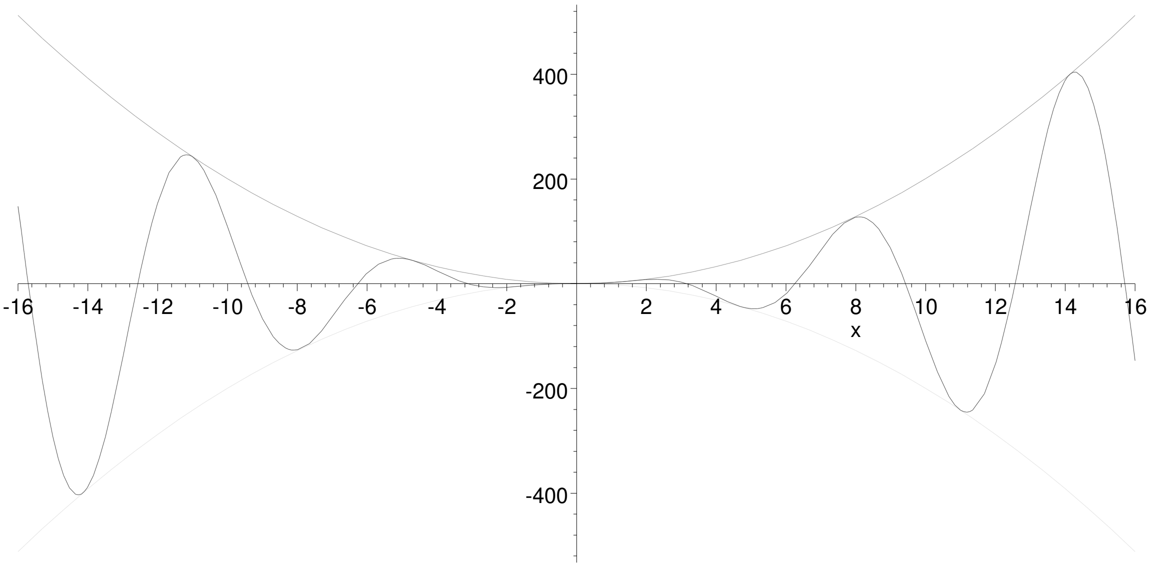
To plot more than one function in a single window, just specify the functions to be plotted as a set. Can you tell which graph is which?

```
> g1:= g1 ;
> p := p ;
```

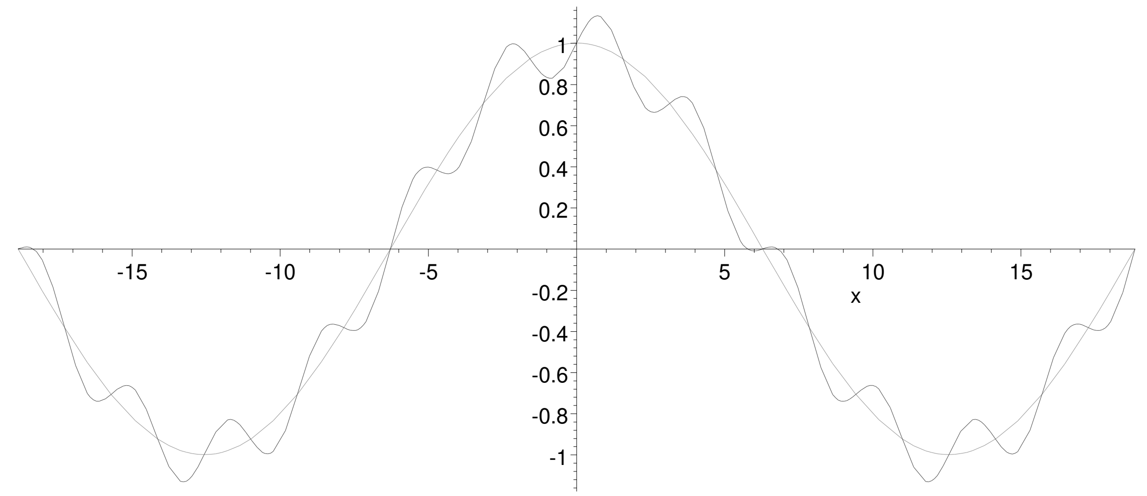
$$g1 := 2\,x^2$$

$$p := 2\sin(x)\,x^2$$

```
> plot( { g1 , - g1 , p } ,  x= -16 ..  16 );
```




```
> h := cos ( (1/4) * x ) :
> f2 := sin( 2  * x ) :
> s:= 0.15 * f2 + h :
> plot( { h , s },  x = - 6* Pi .. 6 * Pi ) ;
```



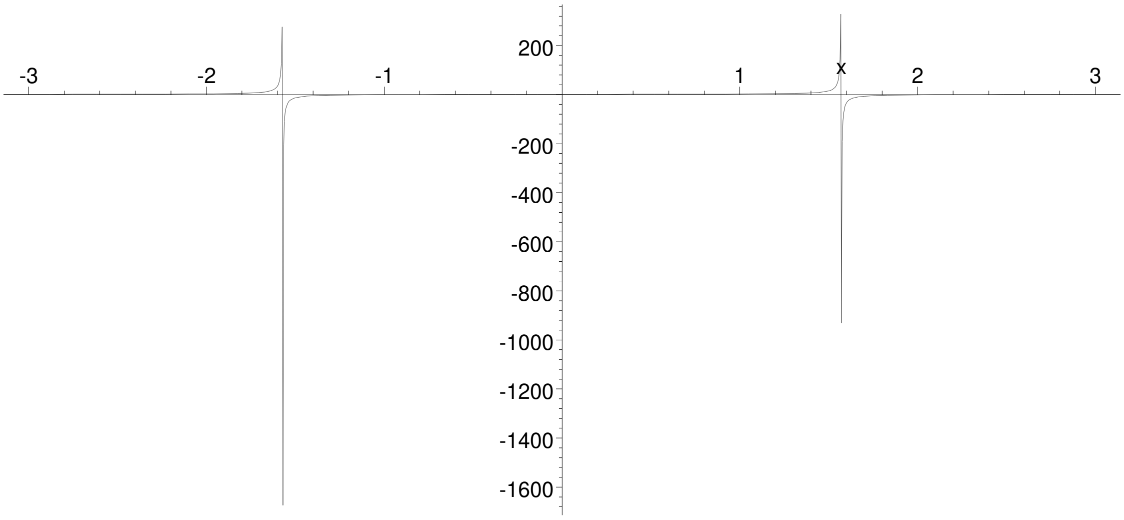
A discontinuous function

The graph of the tangent function has vertical asymptotes at all odd multiples of $\frac{\pi}{2}$. Let's see how Maple handles this.

```
> f :=  tan(x);
```

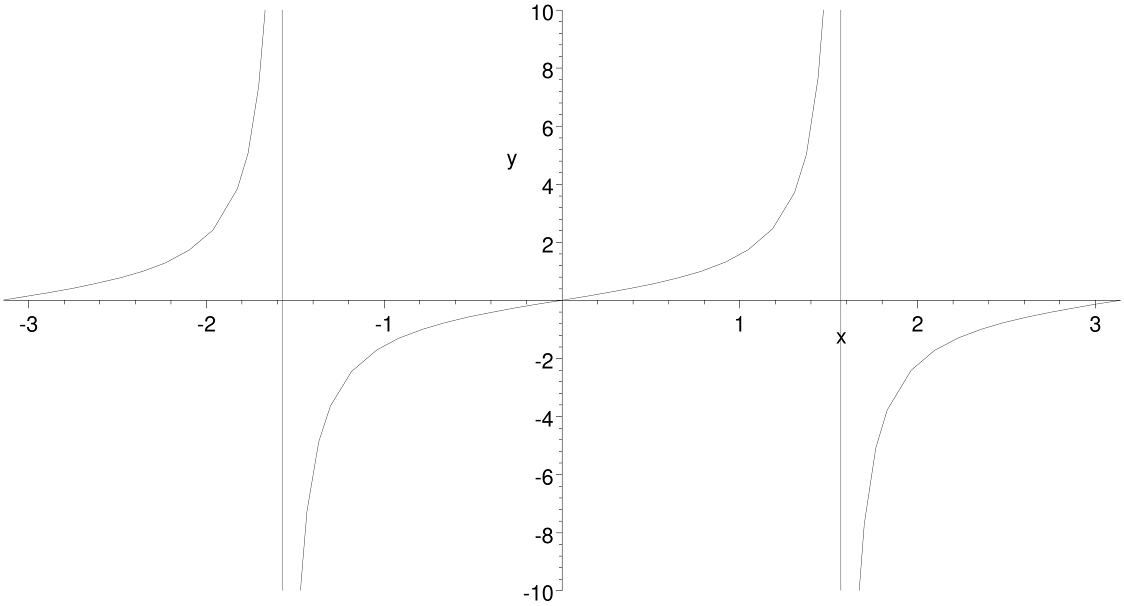
$$f := \tan(x)$$

```
> plot( f , x = -Pi .. Pi );
```

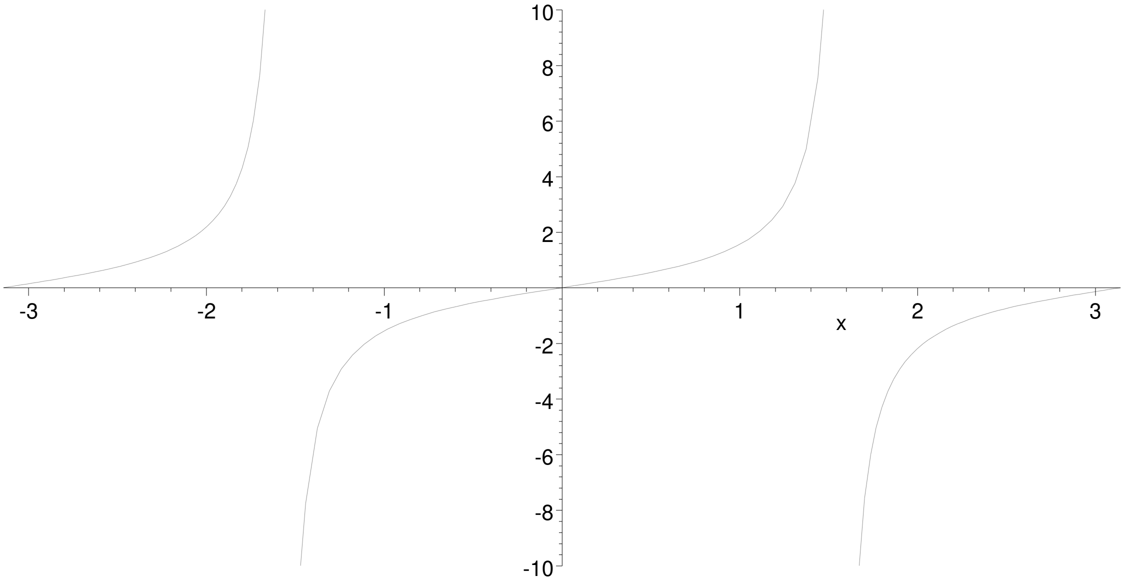


Is this what you expected to see? Why does the graph look like this? What can be done to improve the appearance of the plot?

```
> plot( f,  x = -Pi .. Pi , y =  -10 .. 10);
```

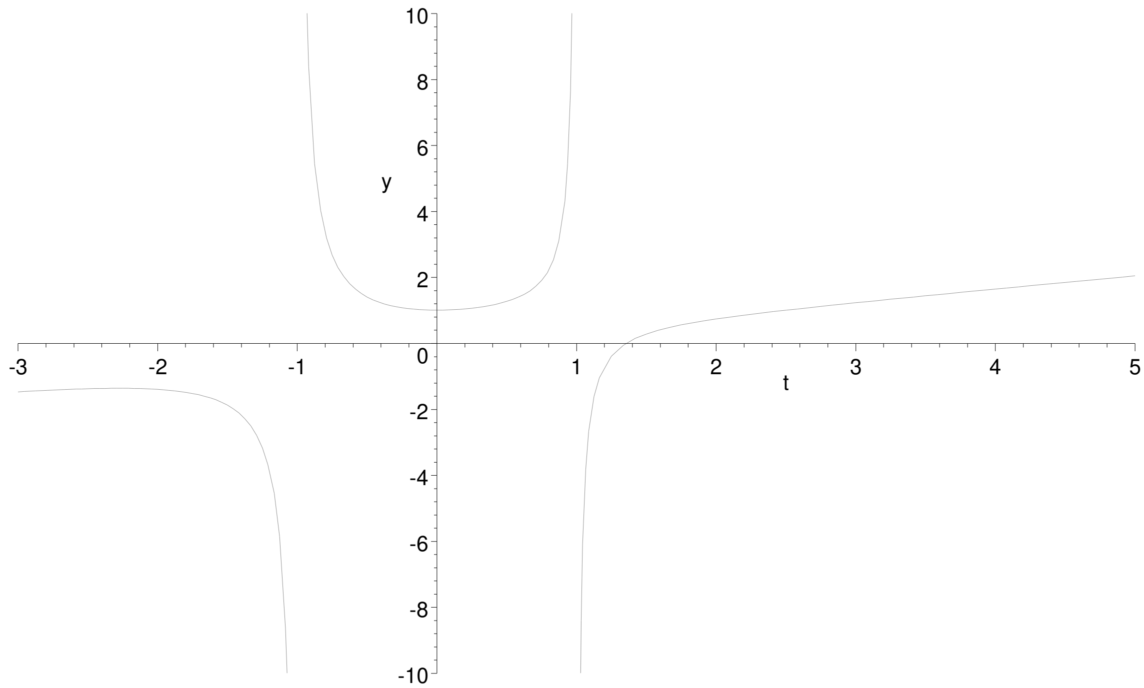


```
> plot( f,  x = -Pi .. Pi ,  -10 .. 10 , discont=true );
```



The key is to restrict the vertical range of the plot and (optionally) tell Maple that the function is discontinuous. This technique is very useful for any function that has vertical asymptotes.

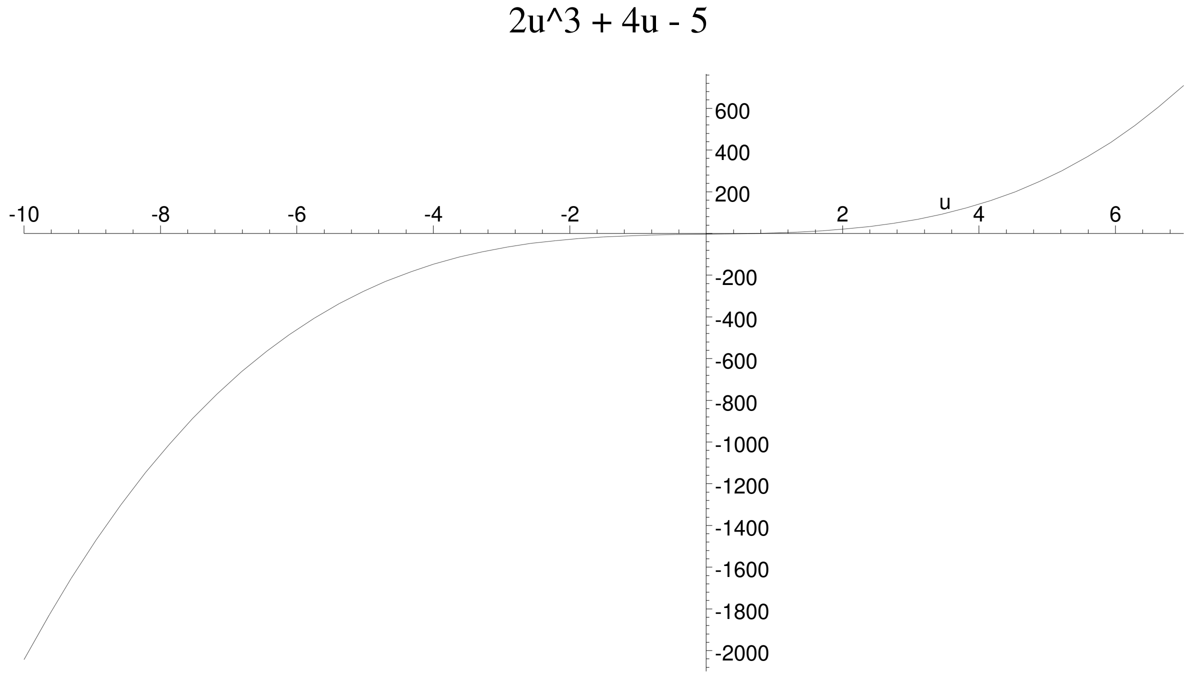
```
> plot ( ( 1 -  0.4*t^3) / ( 1 - t^2 ) , t = -3 .. 5 ,  y = -10 .. 10  ,
  discont = true) ;
```




```
>
```

Graphical method of solving an equation

```
> plot(2 *  u^3 + 4 * u - 5 , u = -10  .. 7 , title = ` 2u^3 + 4u - 5` ) ;
```



Where does the cubic function cross the u-axis? Maybe it would be helpful to cut down the plotting interval from [-10, 7] to [-1, 3], or even narrower, say to [0, 1.5]. Try it. Can you find the u-intercept to 2-decimal point accuracy by this process? It is a powerful method, which we call ZOOMING IN. Maple can also find exact solutions (sometimes, and it can look real messy) with solve, or approximate (floating point) solutions with fsolve.

```
> solve(2 *  u^3 + 4 * u - 5 = 0 , u ) ;
```

$$\frac{1}{6}(270+6\sqrt{2409})^{(1/3)}-4\frac{1}{(270+6\sqrt{2409})^{(1/3)}},-\frac{1}{12}(270+6\sqrt{2409})^{(1/3)}+2\frac{1}{(270+6\sqrt{2409})^{(1/3)}}$$
$$+\frac{1}{2}I\sqrt{3}\left(\frac{1}{6}(270+6\sqrt{2409})^{(1/3)}+4\frac{1}{(270+6\sqrt{2409})^{(1/3)}}\right),-\frac{1}{12}(270+6\sqrt{2409})^{(1/3)}$$
$$+2\frac{1}{(270+6\sqrt{2409})^{(1/3)}}-\frac{1}{2}I\sqrt{3}\left(\frac{1}{6}(270+6\sqrt{2409})^{(1/3)}+4\frac{1}{(270+6\sqrt{2409})^{(1/3)}}\right)$$

```
> evalf( [ % ] );
```

$$[.8934274600, -.4467137300+1.612035816\,I, -.4467137300-1.612035816\,I]$$

```
> fsolve( 2 *  u^3 + 4 * u - 5 = 0 , u ) ;
```

$$.8934274602$$

```
>
```

What about calculus?

Maybe you're wondering if Maple can do derivatives and integrals. It sure can!

```
> f:= 3 * x^ 4 - 2 * x + 5 / x^3 ;
```

$$f := 3\,x^4 - 2\,x + 5\,\frac{1}{x^3}$$

```
> diff( f , x ) ;
```

$$12\,x^3 - 2 - 15\,\frac{1}{x^4}$$

```
> diff( Pi * R^2 * H , R );
```

$$2\,\pi\,R\,H$$

```
>
> F:= t *  sin(t) ;
```

$$F := t\,\sin(t)$$

```
> diff( F  , t );
```

$$\sin(t) + t\,\cos(t)$$

```
>
> easy_integral:= Int( f , x ) ;
```

$$easy\_integral := \int 3\,x^4 - 2\,x + 5\,\frac{1}{x^3}\,dx$$

```
> value( easy_integral ) ;
```

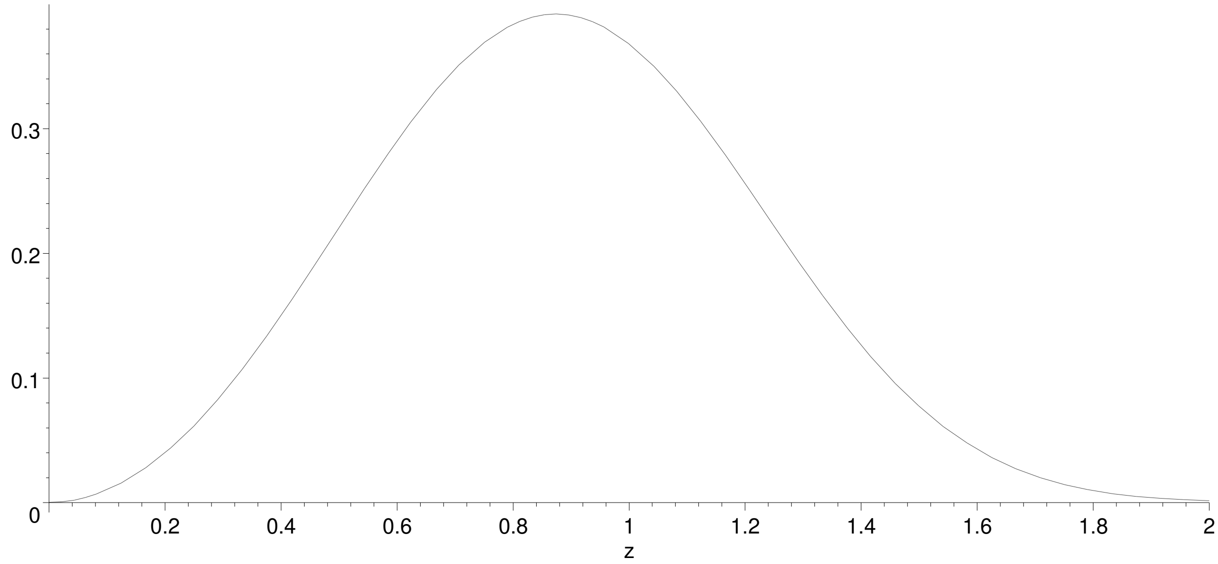
$$\frac{3}{5}\,x^5 - x^2 - \frac{5}{2}\,\frac{1}{x^2}$$

```
>
```

```
> g:= z^2 * exp(- z^3 ) ;
```

$$g := z^2\, \mathbf{e}^{(-z^3)}$$

```
> plot( g , z = 0 .. 2 ) ;
```




```
> harder_integral:= Int( g , z ) ;
```

$$harder\_integral := \int z^2\, \mathbf{e}^{(-z^3)}\, dz$$

```
> value( harder_integral );
```

$$-\frac{1}{3}\, \mathbf{e}^{(-z^3)}$$

```
> definite_integral:= Int( g, z = 0..2 );
```

$$definite\_integral := \int_0^2 z^2\, \mathbf{e}^{(-z^3)}\, dz$$

```
> value( definite_integral );
```

$$-\frac{1}{3}\, \mathbf{e}^{(-8)} + \frac{1}{3}$$

```
> evalf( % );
```

$$.3332215124$$

```
>
```

Congratulations!

```
> restart;
```

You have just completed your first Maple worksheet. Unless you really need the output it is usually a good idea to remove it before saving -- to do this, select Edit, then Remove Output, and finally From Worksheet. If you wish to save a modified version of this worksheet (with or without output), select File in the menu bar, and then either Save (which will destroy the original version of `day1.mws`) or Save As... (which will leave `day1.mws` untouched, and will request a new name for this modified version). Worksheet names should end with `.mws` if created in Maple V Release 5.

```
>
```