Play at the Plate

It's the bottom of the ninth; one out; and the winning run on third when the batter hits a lazy fly ball to left field. The runner leaves third base the instant the leftfielder makes the catch on the run 300 feet from home plate; the outfielder uses 0.6 seconds to release the ball on a direct path to the catcher. The speedy baserunner runs from third to home in 4.0 seconds.

How hard, and with what angle of inclination, must the ball be thrown to prevent the runner from scoring?

— *based on "The Physics of Baseball", Robert K. Adair, 1994 (pp. 58–61).*

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- Assumptions
  - the ball is released from a height of 8 feet above the ground
  - the throw arrives at home plate without bouncing
  - the catcher needs 0.3 seconds to apply the tag
  - the ball weighs approximately 5 oz.
  - $\mathbf{g} = 9.8\mathbf{j}$ m/sec^2
  - magnitude of air resistance is proportional to the square of the speed
  - ignore wind
  - the umpire is in a good position to make the call

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- Mathematical Formulation:

determine $\mathbf{x}'(0)$ and $\|\mathbf{x}'(0)\|$ where $\mathbf{x}$ is a solution to

$$\begin{aligned} m\mathbf{x}'' &= -k\|\mathbf{x}'\|\mathbf{x}' - m\mathbf{g} \\ \mathbf{x}(0) &= 300\mathbf{i} + 8\mathbf{j} \\ \mathbf{x}(3.1) &= \mathbf{0} \end{aligned}$$

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- Typical Results

Arm Strength	Initial Velocity	Release Angle	Flight Time	Outcome
weak	90 mph	27.5°	3.85 sec	
average	100 mph	18°	3.1 sec	
strong	110 mph	13°	2.65 sec	

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- Further Discussion
  - Why does the fielder try to make the catch while running towards home plate?
  - Are there more efficient ways of getting the ball to the catcher?
  - Suppose each outfielder's maximum initial throwing velocity is known. Where should the players be positioned?