```
> restart;
  with( linalg ):
  with( DEtools ):
  with( plots ):
Warning, new definition for norm
Warning, new definition for trace
```

Maple Worksheet to Illustrate Different Qualitative Behaviors of Linear Systems

Prepared by Douglas B. Meade (meade@math.sc.edu)
20 November 1995

Based on a homework exercise in the Boston University Differential Equations Project (Blanchard, Devaney, Hall)

The basic idea is to look at the direction field, and eight solution curves, to a one-parameter family of linear 2x2 systems of ODEs. The qualitative behavior of the solutions to this ODE change quite noticeably for certain values of the parameter (the bifurcation values). See if you can identify, and explain, the transitions without explicitly computing eigenvalues.

An extended version of this problem would be to (attempt to) modify the matrix so that the system does not exhibit certain behaviors. (How do you know this really happens, and is not just an artifact of the discrete set of parameters that are used to create the animated display?)

Modify these definitions at your own peril

```
> phaseportrait := proc( p )
    local dX, X, SYS;
    global M, IC, OPT;
    X := vector([x(t),y(t)]);
    dX := map( diff, evalm(X), t );
    SYS := convert( evalm(dX-M&*X), set );
    DEtools[DEplot]( subs(A=p, SYS ), [x(t),y(t)], -5..5, IC,
                     scene=[x,y], x=-5..5, y=-5..5, OPT );
  end:
> animatephase := proc( param )
    local a;
    plots[display]( [ seq( phaseportrait(a), a=param ) ],
  insequence=true );
  end:
>
```

Feel free to modify the matrix in the first line of this statement, the initial conditions in the third line, and the ranges in the second and fourth lines. (The processing will be much faster if the arrows=THIN option is removed.)

Define the problem to be analyzed by specifiying the one-parameter family of matrices (the parameter is assumed to be A)

```
> M := matrix( [ [ -1/2, A ], [ 1, -1/2 ] ] );
```

$$M := \begin{bmatrix} \frac{-1}{2} & A \\ 1 & \frac{-1}{2} \end{bmatrix}$$

```
> #M := matrix( [ [ 1/2, A ], [ 1, -1/2 ] ] );   # another nice
  example (use the same parameters, etc.)
```

the initial conditions

```
> IC := { [ 0, i, 0 ] $ i=-2..2, [ 0, 0, i ] $ i=-2..2 };
```

$$IC := \{[0,-2,0],[0,-1,0],[0,0,0],[0,1,0],[0,2,0],[0,0,-2],[0,0,-1],[0,0,1],[0,0,2]\}$$

any optional arguments to the DEplot command

```
> #OPT := arrows=LINE:
> OPT := linecolor=BLUE:
```

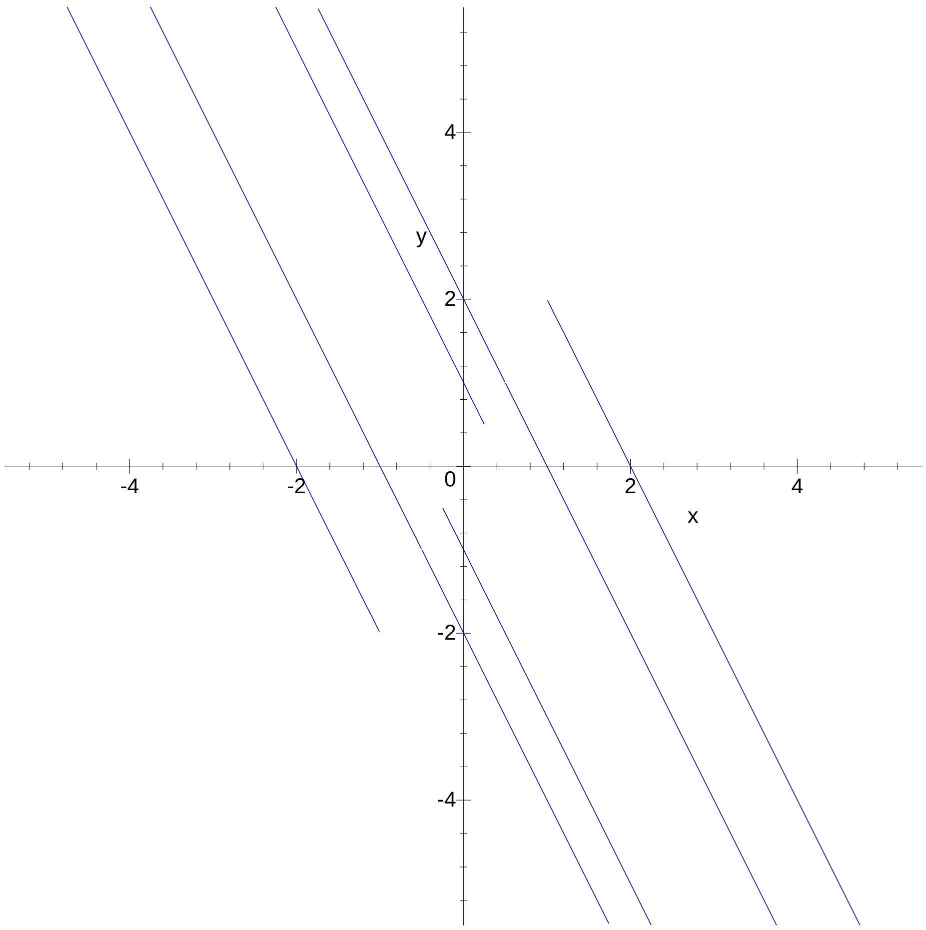
and the values of the parameter which should be assembled to form the animated view of the solutions

```
> PARAM := [ i/20 $ i=-20..20 ];
```

$$PARAM := \left[-1, \frac{-19}{20}, \frac{-9}{10}, \frac{-17}{20}, \frac{-4}{5}, \frac{-3}{4}, \frac{-7}{10}, \frac{-13}{20}, \frac{-3}{5}, \frac{-11}{20}, \frac{-1}{2}, \frac{-9}{20}, \frac{-2}{5}, \frac{-7}{20}, \frac{-3}{10}, \frac{-1}{4}, \frac{-1}{5}, \frac{-3}{20}, \frac{-1}{10}, \frac{-1}{20}, 0, \frac{1}{20}, \frac{1}{10}, \frac{3}{20}, \frac{1}{5}, \frac{1}{4}, \frac{3}{10}, \frac{7}{20}, \frac{2}{5}, \frac{9}{20}, \frac{1}{2}, \frac{11}{20}, \frac{3}{5}, \frac{13}{20}, \frac{7}{10}, \frac{3}{4}, \frac{4}{5}, \frac{17}{20}, \frac{9}{10}, \frac{19}{20}, 1\right]$$

And, finally, we can select either a particular plot

```
> phaseportrait( 1/4 );
```



or an animated sequence of plots

```
> animatephase( PARAM );
```

BE PATIENT! This animation can take several minutes to complete.

```
>
```