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%  sec1-3.ms  -- a Maple worksheet to accompany section 1.3 of Nagle and Saff
%
% Prepared by:  Douglas B. Meade (7 March 1996)
%                    as part of the Maple Supplement for Nagle/Saff (Addison-Wesley)
%
% Updated for Maple V, Release 4, by Douglas B. Meade (5 November 1996)
%
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%
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DIRECTION FIELDS (Section1.3)

```
> restart;
  with( plots ):
  with( DEtools ):
```

Direction fields are a simple, pictorial, technique for understanding how solutions to a differential equation will behave, WITHOUT COMPUTING THE SOLUTION. Direction fields are particularly useful when you first look at a differential equation. Direction fields can be useful when checking if a solution to a differential equation is correct: the tangent lines to the solution must be parallel to the arrows in the direction field. (This works both for symbolic solutions and for numeric solutions.)

Objectives:

- understand the graphical information conveyed by a differential equation
- begin to understand the concept of a solution to a differential equation
- illustrate the use of Maple's *DEplot* command

New Maple Commands:

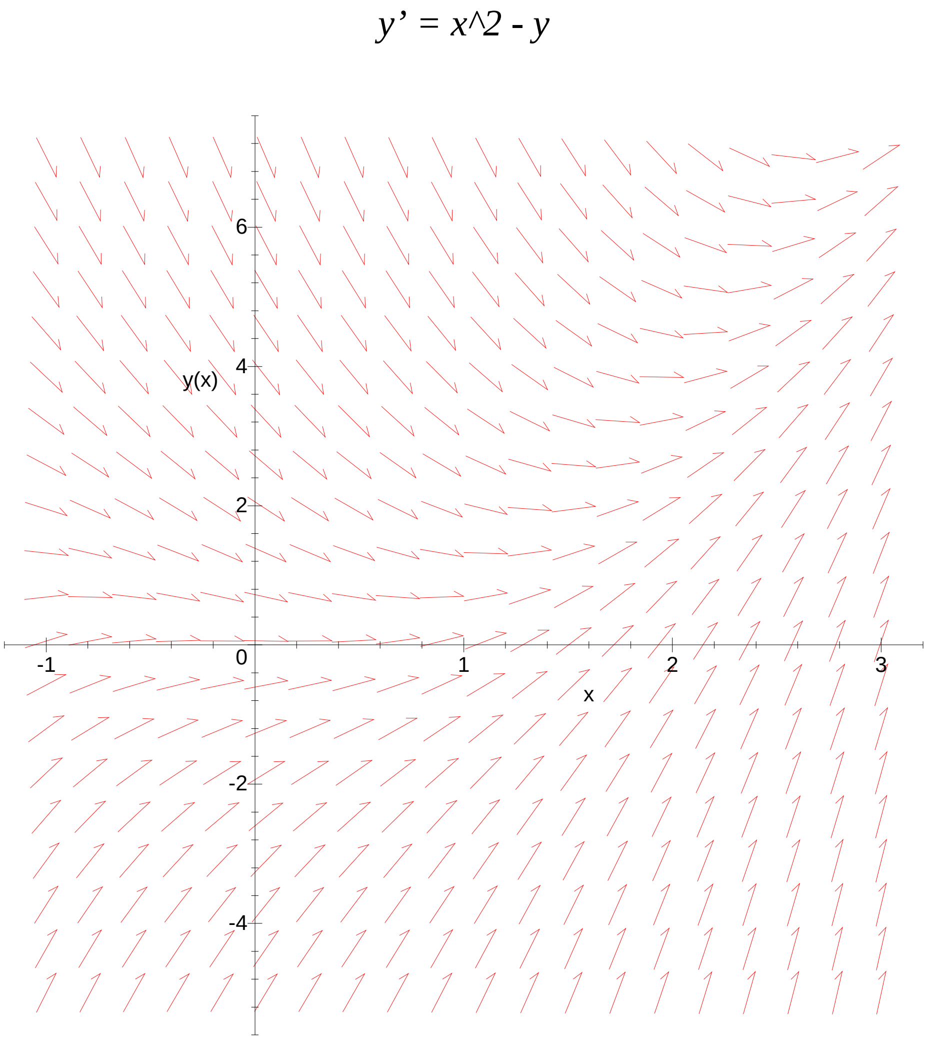
DEplot produces a plot of a direction field for an ODE

The *DEplot* command, in the DEtools package, can be used to produce different types of plots for solutions of differential equations. Our most common use will be to create a plot of the direction field for an ODE. Here is how that might look:

```
> ODE := diff( y(x), x ) = x^2 - y(x);
```

$$ODE := \frac{\partial}{\partial x} y(x) = x^2 - y(x)$$

```
> DEplot( ODE, [y(x)], x=-1..3, y=-5..7, arrows=SMALL, title='y' = x^2 - y' );
```

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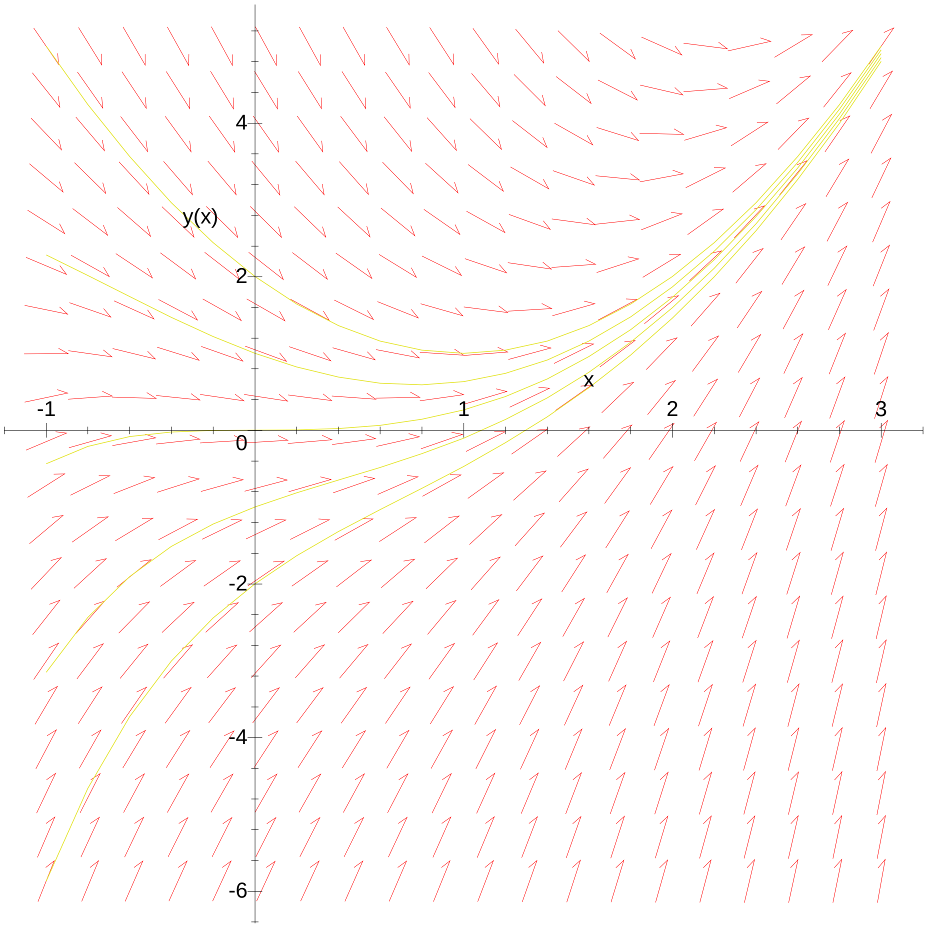
What do you see in this plot?
What do the arrows indicate?
How would a stick float down a stream with these flow patterns?
Does this depend upon where the stick enters the flow?

One way to answer the last question is to use *DEplot* with slightly different arguments.

```
> DEplot( ODE, [y(x)], x=-1..3, { [0,-2], [0,-1], [0,0], [0,1],
  [0,2] },
          arrows=SMALL, title=‘y’ = x^2 - y‘ );
```

y’ = x^2 - y



Notice that the only change is the replacement of the range for the dependent variable (y) with a set of five points. Notice that each of the five curves in the plot passes through a different point in this set.

Do you need to reconsider any of your answers to the previous questions?
Can you now add more detail to your answers?

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Consider the equation y' = -y/x :

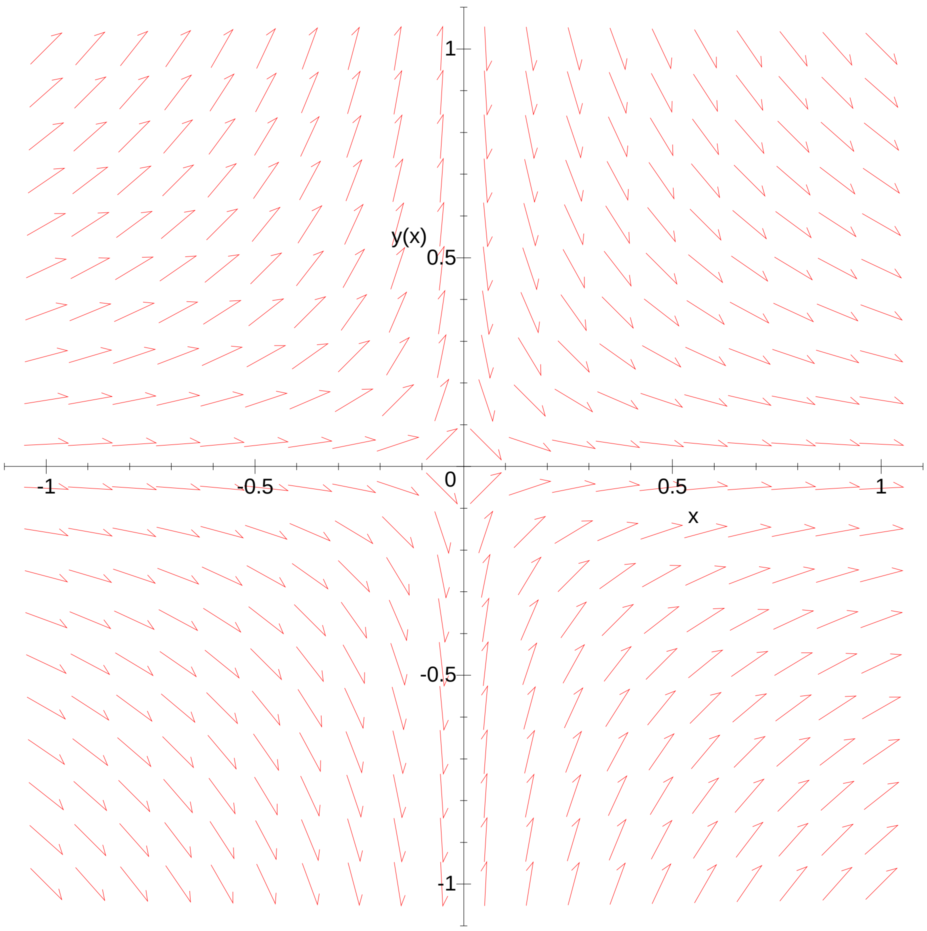
```
> ODE := diff( y(x), x ) = -y(x)/x;
```

$$ODE := \frac{\partial}{\partial x} y(x) = -\frac{y(x)}{x}$$

Does this differential equation satisfy the continuity conditions necessary to apply the Existence and Uniqueness Theorem (Section 1.2)? Explain. Can this be seen in the direction field?

```
> DEplot( ODE, [y(x)], x=-1..1, y=-1..1, arrows=SMALL, title=`y' =
  -y/x` );
```

$y' = -y/x$



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EXAMPLE 1 (p. 19)

The logistic equation for a population with size p (in thousands) at time t is

```
> ODE := diff( p(t), t ) = p(t) * ( 2 - p(t) );
```

$$ODE := \frac{\partial}{\partial t} p(t) = p(t)\,(2 - p(t))$$

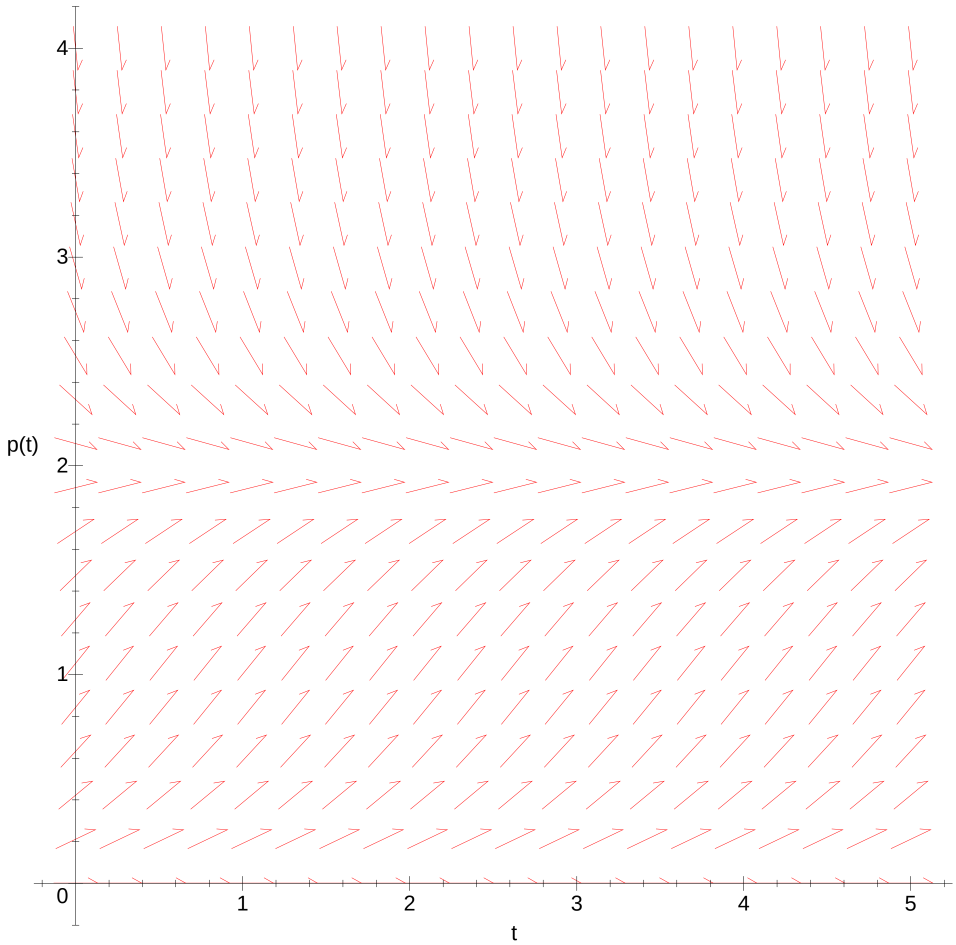
Use a direction field to answer the following questions:

If the initial population is 3000 (that is, p(0)=3), what can you say about the limiting population limit(p(t), t=infinity)?

Can a population of 1000 ever decline to 500? (vice versa?)
Can a population of 1000 ever increase to 3000? (vice versa?)

```
> DEplot( ODE, [p(t)], t=0..5, p=0..4, arrows=SMALL,
          title = ‘p’ = p ( 2 - p )‘ );
```

$$p' = p\,(\,2 - p\,)$$

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