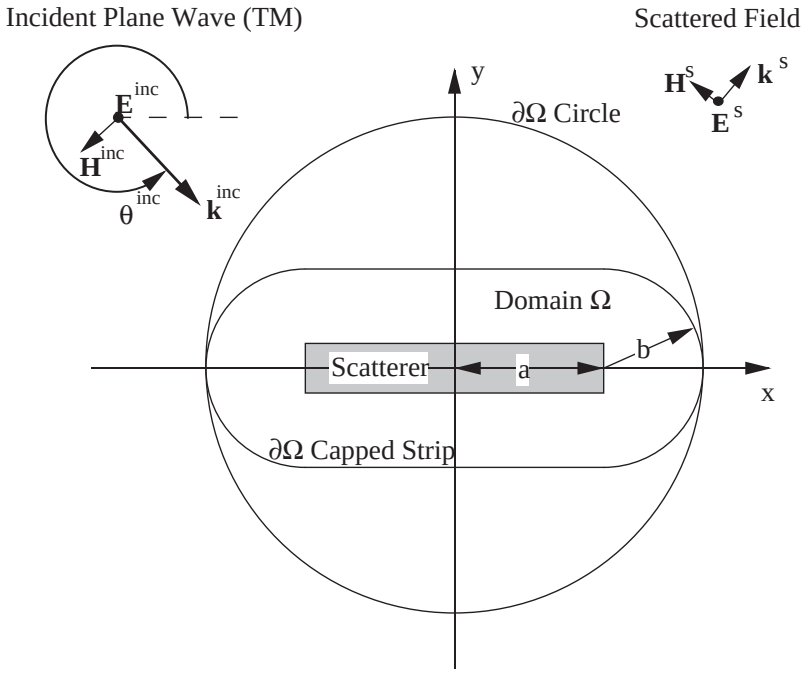
Electromagnetic Scattering in Two Dimensions

Comparison of Boundary Geometries



Scalar Helmholtz Equation: Unbounded Domain

$$
\begin{aligned}
\triangle u + k^2 u &= f && \text{in } \Omega^+ \\
u &= g && \text{on } \Gamma_1 \\
\frac{\partial u}{\partial \rho} - iku &= o(\rho^{-1/2}) && \text{as } \rho \to \infty, \text{ uniformly in } \phi
\end{aligned}
$$

Scattering of a Plane Wave

- Given: monochromatic incident field $u_{\text{inc}}(x,t) = U(x)e^{i\omega t}$
- Solve:

$$\begin{aligned} \triangle u + k^2 u &= 0 && \text{in } \Omega \\ u &= -U && \text{on } \Gamma_1 \\ \frac{\partial u}{\partial n} &= Au + B\frac{\partial u}{\partial t} + D\frac{\partial^2 u}{\partial t^2} + F(U) && \text{on } \Gamma_2 \end{aligned}$$

where

u is the amplitude of the scattered wave ($u(x)e^{i\omega t}$)

$k := \frac{\omega}{c}$ is the wave number

A, B, D are RBC coefficients (depend on x)

Variational Formulation

Find $u \in V_U$ such that $a(u,v) + b(u,v) = f(v)$ for all $v \in V_0$

where

$$
\begin{aligned}
a(u,v) &:= \int_\Omega \nabla u \cdot \nabla v - \int_{\Gamma_2} Auv \\
b(u,v) &:= -k^2 \int_\Omega uv - \int_{\Gamma_2} (B - D_t) u_t v + \int_{\Gamma_2} D u_t v_t \\
f(v) &:= \int_{\Gamma_2} F(U) v \\
V_U &:= \left\{ u \in H^1(\Omega) : u = -U \text{ on } \Gamma_1 \right\}
\end{aligned}
$$

Existence and Uniqueness

Assume

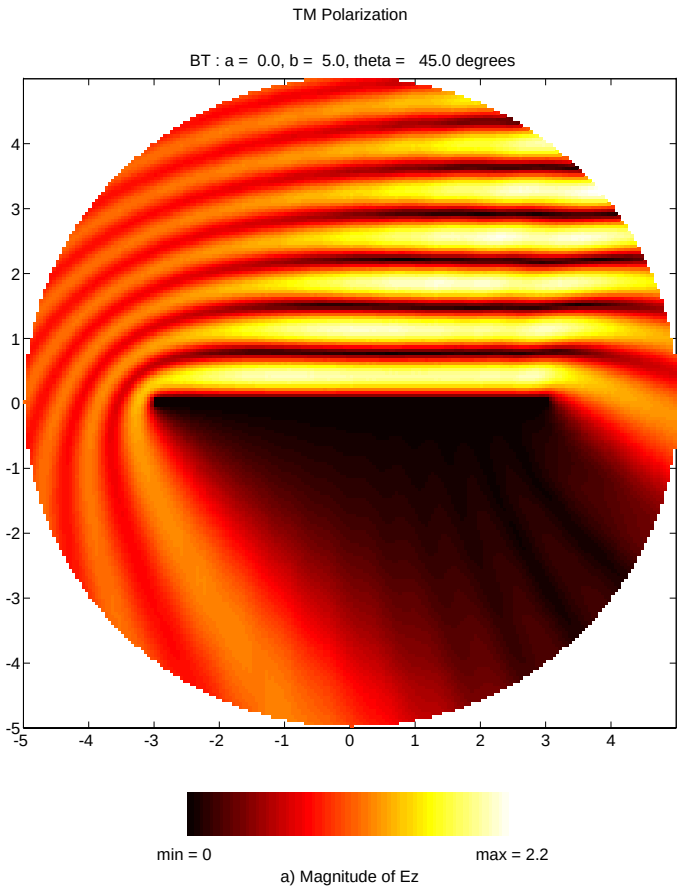
- $\mathrm{Im} A \leq \alpha < 0$ on Γ_2
- necessary smoothness on boundary and coefficients

Then $\exists!$ weak solution in V_U.

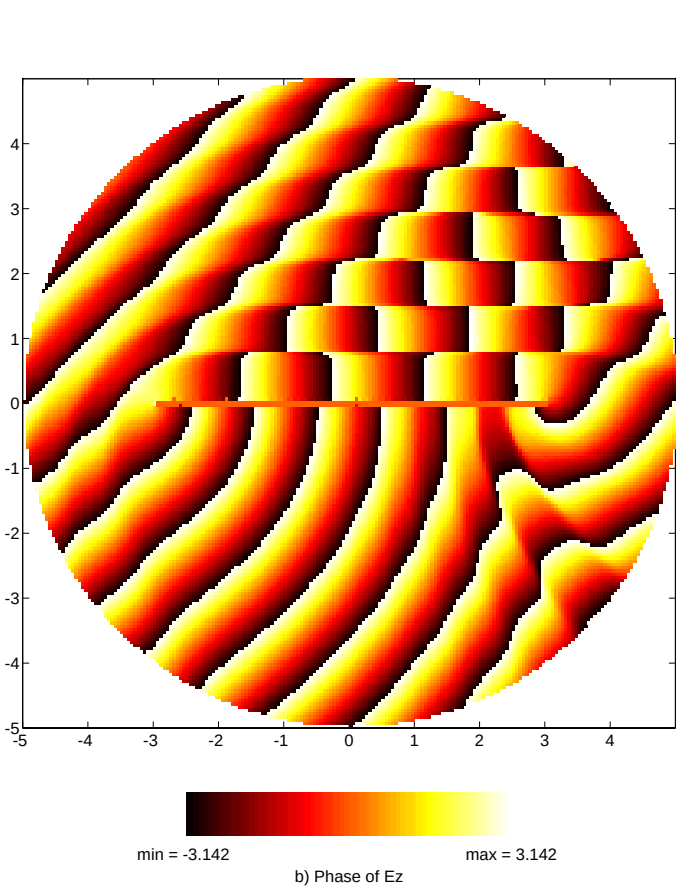
"Proof"

- a is elliptic on V_0
- b is compact
- Fredholm alternative

Numerical Solution: Circular Artificial Boundary

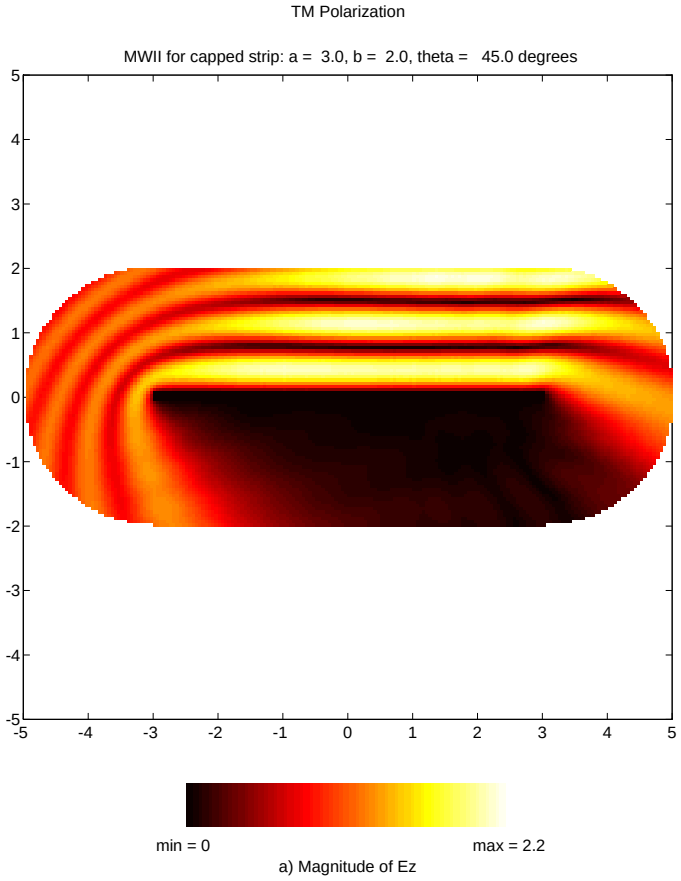


magnitude of E_z

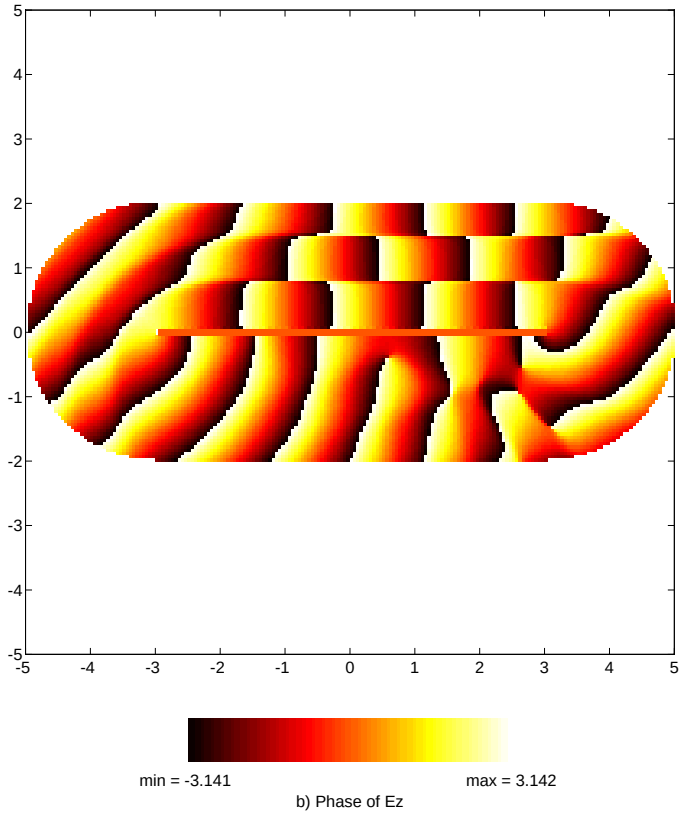


phase of E_z

Numerical Solution: Capped Strip Boundary



magnitude of E_z



phase of E_z

Comparison of Computational Domains

Domain	$\frac{\text{Area}}{\lambda^2}$	Savings	# Node	$\frac{\text{Nodes}}{\lambda}$	$\frac{\text{Nodes}}{\lambda^2}$	# Unknown
Circle (r=5)	77.93	00.0%	37,100	20	476	36,818
CS (b=2.0)	35.97	53.8%	17,342	20	482	17,062
CS (b=1.5)	24.47	68.6%	11,930	20	488	11,650
CS (b=1.0)	14.54	81.3%	7,292	20	502	7,012
CS (b=0.5)	6.19	91.0%	3,344	20	540	3,064

Comparison of Computed Solutions

Domain	Total Time (s)	Time Savings	Relative L_2 Error
Circle (r=5)	2,724.99	0.00%	0.00%
CS (b=2.0)	566.91	79.2%	4.75%
CS (b=1.5)	254.88	90.6%	8.99%
CS (b=1.0)	88.64	96.7%	15.33%
CS (b=0.5)	17.33	99.4%	35.00%