

Contents

Preface	ix
Notes to the Instructor	xi
Overview of Calculus I	1
1 Pre-Calculus Review	3
1.1 Functions	4
1.2 The Basic Functions of Calculus	22
1.3 Building More Functions from Basic Functions	36
1.4 Geometric Series	46
1.5 Logarithms	52
1.S Chapter Summary	58
C.1 Graphs Tell It All	65
C.2 Where Does All That Money Come From?	69
2 Introduction to Calculus	71
2.1 Slope at a Point on a Curve	72
2.2 Four Special Limits	81
2.3 The Limit of a Function: The General Case	98
2.4 Continuous Functions	113
2.5 Three Important Properties of Continuous Functions	127
2.6 Techniques for Graphing	138
2.S Chapter Summary	154
C.3 Bank Interest and the Annual Percentage Yield	161
3 The Derivative	165
3.1 Velocity and Slope: Two Problems with One Theme	166
3.2 The Derivatives of the Basic Functions	181
3.3 Shortcuts for Computing Derivatives	191
3.4 The Chain Rule	207
3.5 Derivative of an Inverse Function	217
3.6 Antiderivatives and Slope Fields	231
3.7 Motion and the Second Derivative	240
3.8 Precise Definition of Limits at Infinity: $\lim_{x \rightarrow \infty} f(x) = L$	249

3.9	Precise Definition of Limits at a Finite Point: $\lim_{x \rightarrow a} f(x) = L$	260
3.S	Chapter Summary	269
C.4	Solar Cookers	279
4	Derivatives and Curve Sketching	283
4.1	Three Theorems about the Derivative	284
4.2	The First-Derivative and Graphing	302
4.3	The Second Derivative and Graphing	312
4.4	Proofs of the Three Theorems	323
4.S	Chapter Summary	328
C.5	Calculus Reassures a Bicyclist	338
C.6	Graphs in Economics	340
5	More Applications of Derivatives	343
5.1	Applied Maximum and Minimum Problems	344
5.2	Implicit Differentiation and Related Rates	366
5.3	Higher Derivatives and the Growth of a Function	386
5.4	Taylor Polynomials and Their Errors	398
5.5	L'Hôpital's Rule for Finding Certain Limits	413
5.6	Natural Growth and Decay	429
5.7	The Hyperbolic Functions and Their Inverses	444
5.S	Chapter Summary	452
C.7	The Uniform Sprinkler	472
6	The Definite Integral	475
6.1	Three Problems That Are One Problem	476
6.2	The Definite Integral	491
6.3	Properties of the Antiderivative and the Definite Integral	511
6.4	The Fundamental Theorem of Calculus	530
6.5	Estimating a Definite Integral	549
6.S	Chapter Summary	569
C.8	Peak Oil Production	584
	Summary of Calculus I	588
	Long Road to Calculus	590
	Overview of Calculus II	594
7	Applications of the Definite Integral	595
7.1	Computing Area by Parallel Cross-Sections	596
7.2	Some Pointers on Drawing	608
7.3	Setting Up a Definite Integral	615
7.4	Computing Volumes by Parallel Cross-Sections	628
7.5	Computing Volumes by Shells	640
7.6	Water Pressure Against a Flat Surface	649

CONTENTS

7.7	Work	656
7.8	Improper Integrals	662
7.S	Chapter Summary	675
C.9	Escape Velocity	680
C.10	Average Speed and Class Size	683
8	Computing Antiderivatives	687
8.1	Shortcuts, Tables, and Technology	689
8.2	The Substitution Method	699
8.3	Integration by Parts	710
8.4	Integrating Rational Functions: The Algebra	726
8.5	Special Techniques	740
8.6	What to do When Confronted with an Integral	754
8.S	Chapter Summary	766
C.11	An Improper Integral in Economics	786
9	Polar Coordinates and Plane Curves	789
9.1	Polar Coordinates	790
9.2	Computing Area in Polar Coordinates	801
9.3	Parametric Equations	809
9.4	Arc Length and Speed on a Curve	819
9.5	The Area of a Surface of Revolution	832
9.6	Curvature	844
9.S	Chapter Summary	856
C.12	The Mercator Map	861
10	Sequences and Their Applications	865
10.1	Introduction to Sequences	867
10.2	Recursively-Defined Sequences and Fixed Points	879
10.3	Bisection Method for Solving $f(x) = 0$	889
10.4	Newton's Method for Solving $f(x) = 0$	899
10.S	Chapter Summary	912
C.13	Hubbert's Peak	918
11	Series	921
11.1	Informal Introduction to Series	923
11.2	Series	932
11.3	The Integral Test	947
11.4	The Comparison Tests	957
11.5	Ratio Tests	966
11.6	Tests for Series with Both Positive and Negative Terms	973
11.S	Chapter Summary	988
C.14	$E = mc^2$	994

12 Applications of Series	997
12.1 Taylor Series	998
12.2 Two Applications of Taylor Series	1008
12.3 Power Series and Their Interval of Convergence	1015
12.4 Manipulating Power Series	1028
12.5 Complex Numbers	1042
12.6 The Relation Between the Exponential and the Trigonometric Functions	1061
12.7 Fourier Series	1071
12.S Chapter Summary	1085
C.15 Sparse Traffic	1089
13 Introduction to Differential Equations	1099
13.1 Modeling and Differential Equations	1100
13.2 Using Slope Fields to Analyze Differential Equations	1104
13.3 Separable Differential Equations	1106
13.4 Euler's Method	1108
13.5 Numerical Solutions to Differential Equations	1110
13.6 Picard's Method	1112
13.S Chapter Summary	1114
Summary of Calculus II	1115
Overview of Calculus III	1116
14 Vectors	1117
14.1 The Algebra of Vectors	1118
14.2 The Dot Product of Two Vectors	1134
14.3 The Cross Product of Two Vectors	1152
14.4 Lines, Planes and Components	1165
14.S Chapter Summary	1185
C.16 Space Flight: The Gravitational Slingshot	1188
C.17 How to Find Planets around Stars	1191
15 Derivatives and Integrals of Vector Functions	1195
15.1 The Derivative of a Vector Function: Velocity and Acceleration	1196
15.2 Curvature and Components of Acceleration	1210
15.3 Line Integrals and Conservative Vector Fields	1222
15.4 Four Applications of Line Integrals	1233
15.S Chapter Summary	1247
C.18 Newton's Law Implies Kepler's Three Laws	1252
C.19 The Suspension Bridge and the Hanging Cable	1261
C.20 The Path of the Rear Wheel of a Scooter	1264

16 Partial Derivatives	1271
16.1 Picturing a Function of Several Variables	1272
16.2 The Many Derivatives of $f(x, y)$	1280
16.3 Change and the Chain Rule	1292
16.4 Directional Derivatives and the Gradient	1307
16.5 Normals and Tangent Planes	1321
16.6 Critical Points and Extrema	1333
16.7 Lagrange Multipliers	1352
16.8 What Everyone Who Will Study Thermodynamics Needs to Know	1364
16.S Chapter Summary	1374
C.21 The Wave in a Rope	1379
17 Plane and Solid Integrals	1383
17.1 The Double Integral: Integrals Over Plane Areas	1385
17.2 Computing $\int_R f(P) dA$ Using Rectangular Coordinates	1399
17.3 Computing $\int_R f(P) dA$ Using Polar Coordinates	1412
17.4 The Triple Integral: Integrals Over Solid Regions	1428
17.5 Cylindrical and Spherical Coordinates	1440
17.6 Iterated integrals for $\int_R f(P) dV$ in Cylindrical or Spherical Coordinates	1454
17.7 Integrals Over Surfaces	1467
17.8 Magnification, Jacobian, and Change of Coordinates	1480
17.9 Moments, Centers of Mass, and Centroids	1485
17.S Chapter Summary	1504
C.22 Solving the Wave Equation	1509
18 The Theorems of Green, Stokes, and Gauss	1513
18.1 Conservative Vector Fields	1515
18.2 Green's Theorem and Circulation	1533
18.3 Green's Theorem, Flux, and Divergence	1547
18.4 Central Fields and Steradians	1560
18.5 The Divergence Theorem in Space (Gauss' Theorem)	1574
18.6 Stokes' Theorem	1585
18.7 Connections Between the Electric Field and $\hat{\mathbf{r}}/\ \mathbf{r}\ ^2$	1603
18.8 Expressing Vector Functions in Other Coordinate Systems	1615
18.9 Maxwell's Equations	1628
18.S Chapter Summary	1633
C.23 How Maxwell Did It	1638
C.24 Heating and Cooling	1642
Summary of Calculus III	1644
A Real Numbers	1645
B Graphs and Lines	1647

C	Topics in Algebra	1651
D	Exponentials (and Logarithms)	1653
E	Trigonometry	1655
F	Logarithms and Exponentials Defined Through Calculus	1657
G	Determinants	1659
H	Jacobian and Change of Coordinates for Multiple Integrals	1661
I	Taylor Series for $f(x, y)$	1663
J	Parameterized Surfaces	1665
K	The Interchange of Limits	1667