
DERIVATIVES

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\ln|x|) = \frac{1}{x}$
- $\frac{d}{dx}(\sin(x)) = \cos(x)$
- $\frac{d}{dx}(\cos(x)) = -\sin(x)$
- $\frac{d}{dx}(\tan(x)) = \sec^2(x)$
- $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$
- $\frac{d}{dx}(\cot(x)) = -\csc^2(x)$
- $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$
- $\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$
- $\frac{d}{dx}(\operatorname{arcsec}(x)) = \frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(a^x) = a^x(\ln(a))$
- $\frac{d}{dx}(\sinh(x)) = \cosh(x)$
- $\frac{d}{dx}(\cosh(x)) = \sinh(x)$

ANTIDERIVATIVES

- $\int x^n dx = \frac{1}{n+1}x^{n+1} \quad n \neq -1$
 $\int \frac{dx}{x} = \ln(x), x > 0 \quad \text{or} \quad \ln|x|, x \neq 0$
- $\int e^x dx = e^x$
- $\int \sin(x) dx = -\cos(x)$
- $\int \cos(x) dx = \sin(x)$
- $\int \tan(x) dx = \ln|\sec(x)| = -\ln|\cos(x)|$
- $\int \cot(x) dx = \ln|\sin(x)| = -\ln|\csc(x)|$
- $\int \sec(x) dx = \ln|\sec(x) + \tan(x)| = \ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|$
- $\int \csc(x) dx = \ln|\csc(x) - \cot(x)| = \ln\left|\tan\left(\frac{x}{2}\right)\right|$
- $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan\left(\frac{x}{a}\right)$

$$10. \int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{1}{a} \arcsin\left(\frac{x}{a}\right), a > 0$$

$$11. \int \frac{dx}{|x|\sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{arcsec}\left(\frac{x}{a}\right)$$

Expressions Containing $ax + b$

$$12. \int (ax + b)^n dx = \frac{1}{a(n+1)}(ax + b)^{n+1}$$

$$13. \int \frac{dx}{ax + b} = \frac{1}{a} \ln|ax + b|$$

$$14. \int \frac{dx}{(ax + b)^2} = \frac{-1}{a(ax + b)}$$

$$15. \int \frac{x dx}{(ax + b)^2} = \frac{b}{a^2(ax + b)} + \frac{1}{a^2} \ln|ax + b|$$

$$16. \int \frac{dx}{x(ax + b)} = \frac{1}{b} \ln\left|\frac{x}{ax + b}\right|$$

$$17. \int \frac{dx}{x^2(ax + b)} = \frac{-1}{bx} + \frac{a}{b^2} \ln\left|\frac{ax + b}{x}\right|$$

$$18. \int \sqrt{ax + b} dx = \frac{2}{3a} \sqrt{(ax + b)^3}$$

$$19. \int x\sqrt{ax + b} dx = \frac{2(3ax - 2b)}{15a^2} \sqrt{(ax + b)^3}$$

$$20. \int \frac{dx}{\sqrt{ax + b}} = \frac{2}{a} \sqrt{ax + b}$$

$$21. \int \frac{\sqrt{ax + b}}{x} dx = 2\sqrt{ax + b} + b \int \frac{dx}{x\sqrt{ax + b}}$$

$$22. \int \frac{dx}{x\sqrt{ax + b}} = \frac{1}{\sqrt{b}} \ln\left|\frac{\sqrt{ax + b} - \sqrt{b}}{\sqrt{ax + b} + \sqrt{b}}\right|, b > 0$$

$$23. \int \frac{dx}{x\sqrt{ax + b}} = \frac{2}{\sqrt{-b}} \arctan \sqrt{\frac{ax + b}{-b}}, b < 0$$

$$24. \int \frac{dx}{x^2\sqrt{ax + b}} = \frac{-\sqrt{ax + b}}{bx} - \frac{a}{2b} \int \frac{dx}{x\sqrt{ax + b}}$$

$$25. \int \sqrt{\frac{cx + d}{ax + b}} dx = \frac{\sqrt{ax + b}\sqrt{cx + d}}{a} + \frac{ad - bc}{2a} \int \frac{dx}{\sqrt{ax + b}\sqrt{cx + d}}$$

Expressions Containing $ax^2 + c$, $x^2 \pm p^2$, and $p^2 - x^2$, $p > 0$

$$26. \int \frac{dx}{p^2 - x^2} = \frac{1}{2p} \ln \left| \frac{p+x}{p-x} \right|$$

$$27. \int \frac{dx}{ax^2 + c} = \begin{cases} \frac{1}{\sqrt{ac}} \arctan \left(x \sqrt{\frac{a}{c}} \right) & a > 0, c > 0 \\ \frac{1}{2\sqrt{-ac}} \ln \left| \frac{x\sqrt{a}-\sqrt{-c}}{x\sqrt{a}+\sqrt{-c}} \right| & a > 0, c < 0 \\ \frac{1}{2\sqrt{-ac}} \ln \left| \frac{\sqrt{c+x\sqrt{-a}}}{\sqrt{c-x\sqrt{-a}}} \right| & a < 0, c > 0 \end{cases}$$

$$28. \int \frac{dx}{(ax^2 + c)^n} = \frac{1}{2(n-1)c} \frac{x}{(ax^2 + c)^{n-1}} + \frac{2n-3}{2(n-1)c} \int \frac{dx}{(ax^2 + c)^{n-1}} \quad n > 1$$

$$29. \int x(ax^2 + c)^n dx = \frac{1}{2a} \frac{(ax^2 + c)^{n+1}}{n+1} \quad n \neq 1$$

$$30. \int \frac{x}{ax^2 + c} dx = \frac{1}{2a} \ln |ax^2 + c|$$

$$31. \int \sqrt{x^2 \pm p^2} dx = \frac{1}{2} \left(x \sqrt{x^2 \pm p^2} \pm p^2 \ln \left| x + \sqrt{x^2 \pm p^2} \right| \right)$$

$$32. \int \sqrt{p^2 - x^2} dx = \frac{1}{2} \left(x \sqrt{p^2 - x^2} + p^2 \arcsin \left(\frac{x}{p} \right) \right)$$

$$33. \int \frac{dx}{\sqrt{x^2 \pm p^2}} = \ln \left| x + \sqrt{x^2 \pm p^2} \right|$$

$$34. \int (p^2 - x^2)^{3/2} dx = \frac{x}{4} (p^2 - x^2)^{3/2} + \frac{3p^2 x}{8} \sqrt{p^2 - x^2} + \frac{3p^4}{8} \arcsin \left(\frac{x}{p} \right)$$

Expressions Containing $ax^2 + bx + c$

$$35. \int \frac{dx}{ax^2 + bx + c} = \begin{cases} \frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \right| & b^2 > 4ac \\ \frac{2}{\sqrt{4ac-b^2}} \arctan \left(\frac{2ax+b}{\sqrt{4ac-b^2}} \right) & b^2 < 4ac \\ \frac{-2}{2ax+b} & b^2 = 4ac \end{cases}$$

$$36. \int \frac{dx}{(ax^2 + bx + c)^{n+1}} = \frac{2ax + b}{n(4ac - b^2)(ax^2 + bx + c)^n} + \frac{2(2n-1)a}{n(4ac - b^2)} \int \frac{dx}{(ax^2 + bx + c)^n}$$

$$37. \int \frac{x dx}{ax^2 + bx + c} = \frac{1}{2a} \ln |ax^2 + bx + c| - \frac{b}{2a} \int \frac{dx}{ax^2 + bx + c}$$

$$38. \int \frac{dx}{\sqrt{ax^2 + bx + c}} = \begin{cases} \frac{1}{\sqrt{a}} \ln \left| 2ax + b + 2\sqrt{a}\sqrt{ax^2 + bx + c} \right| & a > 0 \\ \frac{1}{\sqrt{-a}} \arcsin \left(\frac{-2ax-b}{\sqrt{b^2-4ac}} \right) & a < 0 \end{cases}$$

$$39. \int \frac{x dx}{\sqrt{ax^2 + bx + c}} = \frac{\sqrt{ax^2 + bx + c}}{a} - \frac{b}{2a} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$40. \int \sqrt{ax^2 + bx + c} \, dx = \frac{2ax + b}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

Expressions Containing Powers of Trigonometric Functions

$$41. \int \sin^2(ax) \, dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$42. \int \sin^3(ax) \, dx = -\frac{1}{a} \cos(ax) + \frac{1}{3a} \cos^3(ax)$$

$$43. \int \sin^n(ax) \, dx = -\frac{\sin^{(n-1)}(ax) \cos(ax)}{na} + \frac{n-1}{n} \int \sin^{(n-2)}(ax) \, dx, \quad n \geq 2 \text{ positive integer}$$

$$44. \int \cos^2(ax) \, dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$

$$45. \int \cos^3(ax) \, dx = \frac{1}{a} \sin(ax) - \frac{1}{3a} \sin^3(ax)$$

$$46. \int \cos^n(ax) \, dx = \frac{\cos^{(n-1)}(ax) \sin(ax)}{na} + \frac{n-1}{n} \int \cos^{(n-2)}(ax) \, dx, \quad n \geq \text{positive integer}$$

$$47. \int \tan^2(ax) \, dx = \frac{1}{a} \tan(ax) - x$$

$$48. \int \tan^3(ax) \, dx = \frac{1}{2a} \tan^2(ax) + \frac{1}{a} \ln |\cos(ax)|$$

$$49. \int \tan^n(ax) \, dx = \frac{\tan^{(n-1)}(ax)}{a(n-1)} - \int \tan^{(n-2)}(ax) \, dx, \quad n \neq 1$$

$$50. \int \sec^2(ax) \, dx = \frac{1}{a} \tan(ax)$$

$$51. \int \sec^3(ax) \, dx = \frac{1}{2a} \sec(ax) \tan(ax) + \frac{1}{2a} \ln |\sec(ax) + \tan(ax)|$$

$$52. \int \sec^n(ax) \, dx = \frac{\sec^{(n-2)}(ax) \tan(ax)}{a(n-1)} - \frac{n-2}{n-1} \int \sec^{(n-2)}(ax) \, dx, \quad n \neq 1$$

$$53. \int \frac{dx}{1 \pm \sin(ax)} = \mp \frac{1}{a} \tan \left(\frac{\pi}{4} \mp \frac{ax}{2} \right)$$

Expressions Containing Algebraic and Trigonometric Functions

$$54. \int x \sin(ax) \, dx = \frac{1}{a^2} \sin(ax) - \frac{x}{a} \cos(ax)$$

$$55. \int x \cos(ax) \, dx = \frac{1}{a^2} \cos(ax) + \frac{x}{a} \sin(ax)$$

$$56. \int x^n \sin(ax) \, dx = -\frac{1}{a} x^n \cos(ax) + \frac{n}{a} \int x^{n-1} \cos(ax) \, dx \quad n \text{ positive}$$

$$57. \int x^n \cos(ax) dx = \frac{1}{a} x^n \sin(ax) - \frac{n}{a} \int x^{n-1} \sin(ax) dx \quad n \text{ positive}$$

$$58. \int \sin(ax) \cos(bx) dx = \frac{-\cos((a-b)x)}{2(a-b)} - \frac{\cos((a+b)x)}{2(a+b)} \quad a^2 \neq b^2$$

Expressions Containing Exponential and Logarithmic Functions

$$59. \int x e^{ax} dx = \frac{1}{a^2} e^{ax} (ax - 1)$$

$$60. \int x b^{ax} dx = \frac{1}{a^2} \frac{b^{ax}}{(\ln(b))^2} (a \ln(b)x - 1)$$

$$61. \int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

$$62. \int e^{ax} \sin(bx) dx = \frac{e^{ax}}{a^2 + b^2} (a \sin(bx) - b \cos(bx))$$

$$63. \int e^{ax} \cos(bx) dx = \frac{e^{ax}}{a^2 + b^2} (a \cos(bx) + b \sin(bx))$$

$$64. \int \ln(ax) dx = x (\ln(ax) - 1)$$

$$65. \int x^n \ln(ax) dx = x^{n+1} \left(\frac{\ln(ax)}{n+1} - \frac{1}{(n+1)^2} \right) \quad n = 0, 1, 2, \dots$$

$$66. \int (\ln(ax))^2 dx = x^2 ((\ln(ax))^2 - 2 \ln(ax) + 2)$$

Expressions Containing Inverse Trigonometric Functions

$$67. \int \arcsin(ax) dx = x \arcsin(ax) + \frac{1}{a} \sqrt{1 - a^2 x^2}$$

$$68. \int \arccos(ax) dx = x \arccos(ax) - \frac{1}{a} \sqrt{1 - a^2 x^2}$$

$$69. \int \operatorname{arcsec}(ax) dx = x \operatorname{arcsec}(ax) - \frac{1}{a} \ln \left| ax + \sqrt{a^2 x^2 - 1} \right|$$

$$70. \int \operatorname{arccsc}(ax) dx = x \operatorname{arccsc}(ax) + \frac{1}{a} \ln \left| ax + \sqrt{a^2 x^2 - 1} \right|$$

$$71. \int \arctan(ax) dx = x \arctan(ax) - \frac{1}{2a} \ln(1 + a^2 x^2)$$

$$72. \int \operatorname{arccot}(ax) dx = x \operatorname{arccot}(ax) + \frac{1}{2a} \ln(1 + a^2 x^2)$$

Some Special Integrals

$$73. \int_0^{\pi/2} \sin^n(x) dx = \int_0^{\pi/2} \cos^n(x) dx = \begin{cases} \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (n-1)}{2 \cdot 4 \cdot 6 \cdot 8 \cdots (n)} \frac{\pi}{2} & n \text{ even} \\ \frac{2 \cdot 4 \cdot 6 \cdot 8 \cdots (n-1)}{1 \cdot 3 \cdot 5 \cdot 7 \cdots (n)} & n \text{ odd} \end{cases}$$

$$74. \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Calculus

December 4, 2010

Contents

Preface	ix
Notes to the Instructor	xi
Overview of Calculus I	1
1 Pre-Calculus Review	3
1.1 Functions	4
1.2 The Basic Functions of Calculus	22
1.3 Building More Functions from Basic Functions	36
1.4 Geometric Series	46
1.5 Logarithms	51
1.S Chapter Summary	57
C.1 Graphs Tell It All	64
2 Introduction to Calculus	69
2.1 Slope at a Point on a Curve	70
2.2 Four Special Limits	79
2.3 The Limit of a Function: The General Case	96
2.4 Continuous Functions	111
2.5 Three Important Properties of Continuous Functions	125
2.6 Techniques for Graphing	136
2.S Chapter Summary	152
C.2 Bank Interest and the Annual Percentage Yield	159
3 The Derivative	163
3.1 Velocity and Slope: Two Problems with One Theme	164
3.2 The Derivatives of the Basic Functions	179
3.3 Shortcuts for Computing Derivatives	189
3.4 The Chain Rule	205
3.5 Derivative of an Inverse Function	215
3.6 Antiderivatives and Slope Fields	229
3.7 Motion and the Second Derivative	238
3.8 Precise Definition of Limits at Infinity: $\lim_{x \rightarrow \infty} f(x) = L$	247
3.9 Precise Definition of Limits at a Finite Point: $\lim_{x \rightarrow a} f(x) = L$	258

3.S Chapter Summary	267
C.3 Solar Cookers	277
4 Derivatives and Curve Sketching	281
4.1 Three Theorems about the Derivative	282
4.2 The First-Derivative and Graphing	300
4.3 The Second Derivative and Graphing	310
4.4 Proofs of the Three Theorems	321
4.S Chapter Summary	326
C.4 Calculus Reassures a Bicyclist	336
C.5 Graphs in Economics	338
5 More Applications of Derivatives	341
5.1 Applied Maximum and Minimum Problems	342
5.2 Implicit Differentiation and Related Rates	364
5.3 Higher Derivatives and the Growth of a Function	384
5.4 Taylor Polynomials and Their Errors	396
5.5 L'Hôpital's Rule for Finding Certain Limits	411
5.6 Natural Growth and Decay	427
5.7 The Hyperbolic Functions and Their Inverses	442
5.S Chapter Summary	450
C.6 The Uniform Sprinkler	470
6 The Definite Integral	473
6.1 Three Problems That Are One Problem	474
6.2 The Definite Integral	489
6.3 Properties of the Antiderivative and the Definite Integral	509
6.4 The Fundamental Theorem of Calculus	528
6.5 Estimating a Definite Integral	547
6.S Chapter Summary	567
C.7 Peak Oil Production	582
Summary of Calculus I	586
Long Road to Calculus	588
Overview of Calculus II	592
7 Applications of the Definite Integral	593
7.1 Computing Area by Parallel Cross-Sections	594
7.2 Some Pointers on Drawing	606
7.3 Setting Up a Definite Integral	613
7.4 Computing Volumes by Parallel Cross-Sections	626
7.5 Computing Volumes by Shells	638
7.6 Water Pressure Against a Flat Surface	647
7.7 Work	654

7.8	Improper Integrals	660
7.S	Chapter Summary	673
C.8	Escape Velocity	678
C.9	Average Speed and Class Size	681
8	Computing Antiderivatives	685
8.1	Shortcuts, Tables, and Technology	687
8.2	The Substitution Method	697
8.3	Integration by Parts	708
8.4	Integrating Rational Functions: The Algebra	724
8.5	Special Techniques	738
8.6	What to do When Confronted with an Integral	751
8.S	Chapter Summary	763
C.10	An Improper Integral in Economics	783
9	Polar Coordinates and Plane Curves	787
9.1	Polar Coordinates	788
9.2	Computing Area in Polar Coordinates	799
9.3	Parametric Equations	807
9.4	Arc Length and Speed on a Curve	817
9.5	The Area of a Surface of Revolution	830
9.6	Curvature	842
9.S	Chapter Summary	854
C.11	The Mercator Map	859
10	Sequences and Their Applications	863
10.1	Introduction to Sequences	865
10.2	Recursively-Defined Sequences and Fixed Points	877
10.3	Bisection Method for Solving $f(x) = 0$	887
10.4	Newton's Method for Solving $f(x) = 0$	897
10.S	Chapter Summary	910
C.12	Hubbert's Peak	916
11	Series	919
11.1	Informal Introduction to Series	921
11.2	Series	930
11.3	The Integral Test	945
11.4	The Comparison Tests	955
11.5	Ratio Tests	964
11.6	Tests for Series with Both Positive and Negative Terms	971
11.S	Chapter Summary	986
C.13	$E = mc^2$	992

12 Applications of Series	995
12.1 Taylor Series	996
12.2 Two Applications of Taylor Series	1006
12.3 Power Series and Their Interval of Convergence	1013
12.4 Manipulating Power Series	1026
12.5 Complex Numbers	1040
12.6 The Relation Between the Exponential and the Trigonometric Functions	1058
12.7 Fourier Series	1068
12.S Chapter Summary	1082
C.14 Sparse Traffic	1086
C.15 Where Does All That Money Come From?	1095
13 Introduction to Differential Equations	1097
13.1 Modeling and Differential Equations	1098
13.2 Using Slope Fields to Analyze Differential Equations	1102
13.3 Separable Differential Equations	1104
13.4 Euler's Method	1106
13.5 Numerical Solutions to Differential Equations	1108
13.6 Picard's Method	1110
13.S Chapter Summary	1112
Summary of Calculus II	1113
Overview of Calculus III	1114
14 Vectors	1115
14.1 The Algebra of Vectors	1116
14.2 The Dot Product of Two Vectors	1132
14.3 The Cross Product of Two Vectors	1150
14.4 Lines, Planes and Components	1163
14.S Chapter Summary	1183
C.16 Space Flight: The Gravitational Slingshot	1186
C.17 How to Find Planets around Stars	1189
15 Derivatives and Integrals of Vector Functions	1193
15.1 The Derivative of a Vector Function: Velocity and Acceleration	1194
15.2 Curvature and Components of Acceleration	1208
15.3 Line Integrals and Conservative Vector Fields	1220
15.4 Four Applications of Line Integrals	1231
15.S Chapter Summary	1245
C.18 Newton's Law Implies Kepler's Three Laws	1250
C.19 The Suspension Bridge and the Hanging Cable	1259
C.20 The Path of the Rear Wheel of a Scooter	1262

16 Partial Derivatives	1269
16.1 Picturing a Function of Several Variables	1270
16.2 The Many Derivatives of $f(x, y)$	1278
16.3 Change and the Chain Rule	1290
16.4 Directional Derivatives and the Gradient	1305
16.5 Normals and Tangent Planes	1319
16.6 Critical Points and Extrema	1331
16.7 Lagrange Multipliers	1350
16.8 What Everyone Who Will Study Thermodynamics Needs to Know	1362
16.S Chapter Summary	1372
C.21 The Wave in a Rope	1377
17 Plane and Solid Integrals	1381
17.1 The Double Integral: Integrals Over Plane Areas	1383
17.2 Computing $\int_R f(P) dA$ Using Rectangular Coordinates	1397
17.3 Computing $\int_R f(P) dA$ Using Polar Coordinates	1410
17.4 The Triple Integral: Integrals Over Solid Regions	1426
17.5 Cylindrical and Spherical Coordinates	1438
17.6 Iterated integrals for $\int_R f(P) dV$ in Cylindrical or Spherical Coordinates	1452
17.7 Integrals Over Surfaces	1465
17.8 Magnification, Jacobian, and Change of Coordinates	1478
17.9 Moments, Centers of Mass, and Centroids	1483
17.S Chapter Summary	1502
C.22 Solving the Wave Equation	1507
18 The Theorems of Green, Stokes, and Gauss	1511
18.1 Conservative Vector Fields	1513
18.2 Green's Theorem and Circulation	1531
18.3 Green's Theorem, Flux, and Divergence	1545
18.4 Central Fields and Steradians	1558
18.5 The Divergence Theorem in Space (Gauss' Theorem)	1572
18.6 Stokes' Theorem	1583
18.7 Connections Between the Electric Field and $\hat{\mathbf{r}}/\ \mathbf{r}\ ^2$	1601
18.8 Expressing Vector Functions in Other Coordinate Systems	1613
18.9 Maxwell's Equations	1626
18.S Chapter Summary	1631
C.23 How Maxwell Did It	1636
C.24 Heating and Cooling	1640
Summary of Calculus III	1642
A Real Numbers	1643
B Graphs and Lines	1645

C	Topics in Algebra	1649
D	Exponentials (and Logarithms)	1651
E	Trigonometry	1653
F	Logarithms and Exponentials Defined Through Calculus	1655
G	Determinants	1657
H	Jacobian and Change of Coordinates for Multiple Integrals	1659
I	Taylor Series for $f(x, y)$	1661
J	Parameterized Surfaces	1663
K	The Interchange of Limits	1665

Preface

As we wrote each section of this book, we kept in our minds an image of the student who will be using it. The student will be busy, taking other demanding classes besides calculus. Also that student may well need to understand the vector analysis chapter, which represents the culmination of the theory and applications within the covers of this book.

That image shaped both the exposition and the exercises in each section.

A section begins with a brief introduction. Then it quickly moves to an informal presentation of the central idea of the section, followed by examples. After the student has a feel for the core of the section, a formal proof is given.

Those proofs are what hold the course together and serve also as a constant review. For this reason we chose student-friendly proofs, adequately motivated. For instance, instead of the elegant, short proof that absolute convergence of a series implies convergence, we employed a longer, but more revealing proof. We avoid pulling tricks out of thin air; hence our new motivation of the cross product. Where one proof will do, we do not use two. Also, rather than proving the theorem in complete generality, we may treat only a special case, if that case conveys the flow of the general proof.

As we assembled the exercises we labeled them R (routine), M (medium), and C (challenging), to make sure we had enough of each type. The R- exercises focus on definitions and algorithmic calculations. The M-type require more thought. The C-type either demand a deeper understanding or offer an alternative view of the material.

In order to keep the sections as short as feasible, we concentrated on the mathematics. We avoided bringing in too many applications in the text, which would not only make the sections too long to be read by a busy student, but would not do justice to the applications. However, because the applications are the reason most students study the subject, each chapter concludes with a thorough treatment of an application in a section called “Calculus is Everywhere.” Because each stands alone, students and instructors are free to deal with it as they please, depending on time available and interest: skip it, glance at it, browse through it, or read it carefully. The presence of the Calculus is Everywhere sections allowed us to replace exercises that start with a long description of an application and end with a trivial bit of calculus. Our guiding theme is do one thing at a time, whether it’s exposition, an example, or an exercise.

As we worked on each section we asked ourselves several questions: Is it the right length? Does it get to the point quickly? Does it focus on just one idea and correspond to one lecture? Are there enough examples? Are there enough exercises, from routine to challenging?

Curvature is treated twice, first in the plane, without vectors, and later, in space, with vectors. We do this for two reasons. First, it provides the student background for appreciating the vector approach. Second, it reduces the vector treatment section to a reasonable length.

Many students will use vector analysis in engineering and physics courses. One of us sat in on a sophomore level electromagnetic course in order to find out how the concepts were applied and what was expected of the students. That inspired a major revision of that chapter.

In addition, the new edition reaches limits and derivatives as early as possible, and as simply as possible. Also, we introduce the Permanence Property, which implies that a continuous function that is positive at a number remains positive nearby. This is referred to several times; hence we gave it a name.

The controversy about what to do about epsilon-delta proofs will never end. Therefore in our text the

instructor is free to choose what to do about such proofs. To make our treatment student-friendly, we broke it into two sections. The first section treats limits at infinity because the diagrams are easier and the concept is more accessible. The second deals with limits at a number. A rigorous proof is given there of the Permanence Property, illustrating the power of the epsilon-delta approach to demonstrate something that is not intuitively obvious. Later in the book the rigorous approach appears only in some C-level exercises, giving the instructor and student an opportunity to reinforce that approach if they so choose.

Throughout the book we include exercises that ask only for computing a derivative or an integral. These exercises are intended to keep those skills sharp. We do not want to assign to exercises that explore a new concept the additional responsibility of offering extensive practice in calculations. This illustrates our general principle: do only one thing at a time, and do it clearly.

One of our objectives was to develop throughout the chapters the mathematical maturity a student needs to understand the vector analysis in the final chapter. For instance, we often include an exercise which asks the student to state a theorem in their own words without mathematical symbols. We had found while doing some pro-bono tutoring that students do not read a theorem carefully. No wonder they didn't know what to do when a supposedly routine exercise asked them to verify a theorem in a particular case.

Notes to the Instructor

§1.1 A review and a reference. It gets right to the point. The examples provide background for later work. Exercises 35 to 39 bring in the transcendental functions early.

§1.2 Reinforces the exponential and logarithmic functions early and its summary emphasizes the most difficult functions, logarithms. We save “modeling” for later, abiding by our principle, “one section, one main idea.” Exercise 52 asks students to think on their own, to be ready for the last third of the book.

§1.3 Quickly builds all the functions needed. We do this for two reasons: to give the students more time to deal with them and to have them available for examples and exercises.

Following our policy of doing just one thing at a time, we develop limits in Chapter 2, separating them from their application in Chapter 3, which introduces the derivative.

§2.2 Focuses on the basic limits needed in Chapter 3. The binomial theorem is not used because many students are not familiar or comfortable with it.

§2.5 Introduces the Permanence Property, which is used several times in later chapters. Hence, we give it a name.

§2.6 Chapter summaries offer an overall perspective and emphasis not possible in an individual section.

§3.1 Introduces the derivative in the traditional way, by velocity and the tangent line. Because of the earlier development of the key limits, this section can be kept short.

§3.3 By using the Δ -notation, we obtain the derivatives of $f + g$, fg , and f/g without using any “student unfriendly” tricks, such as adding and subtracting $f(x)g(x)$.

§3.4 The rigorous proof of the chain rule is left as an exercise with detailed sketch. That enables the student reading the text to concentrate on learning how to apply the chain rule.

§3.5 Obtains the derivatives of the inverse functions, using the chain rule. There is no need to wait until implicit differentiation is discussed. That way the chapter can focus on obtaining the differentiation formulas. Exercises 76 and 86 are two of the “Sam and Jane” exercises that add a light touch and invite the students to think on their own.

§3.6 Introduces antiderivatives well before the definite integral appears in Chapter 6, so that the two concepts are adequately separated in time. Slope fields will be used later.

§3.7 Note that the higher derivatives will be put to work as early as Section 5.4, which concerns Taylor polynomials.

§3.8 and 3.9 We delayed the precise definitions of limits in order to give the students more time to work with limits before facing these definitions. These sections are optional. Section 3.8 is easier. One may separate the two sections by several days to let the first one sink in. Note that Example 2 in Section 3.9 shows how useful a precise definition is, as it justifies the Permanence Principle.

§3.9 Emphasizes the essentials and invites more practice in differentiation. Throughout the remaining chapters we include exercises on straightforward differentiation.

Chapter 4 Concentrates on just one theme: using f' and f'' to graph a function. This provides a strong foundation for Chapter 5, which includes optimization.

§5.3 Shows how a higher derivative influences the growth of a function and sets the stage for Section 5.4, Taylor polynomials and their errors. The growth theorem of Section 5.3 is used in exercises in Chapter 6 to obtain the error in approximating a definite integral by the trapezoidal or Simpson's methods.

§5.6 Exercise 39 raises interesting questions about exponential growth.

§6.1 This section keeps to a readable length by avoiding involvement with a formula for the sum $1^2 + 2^2 + \cdots + n^2$.

§6.2 Anticipates the formula $F(b) - F(a)$ for evaluating a definite integral.

§6.5 Exercises such as 44 and 45 are not as hard as one would expect, because the steps are outlined. Such exercises review several important concepts.

Overview of Calculus I

There are two main concepts in calculus: the derivative and the integral. Two scenarios that could occur in your car introduce both concepts.

Scenario A

Your speedometer is broken, but your odometer works. Your passenger writes down the odometer reading every second. How could you estimate the speed, which may vary from second to second?

The speedometer measures your current speed. The odometer measures the total distance covered.

This scenario is related to the “derivative,” the key concept of differential calculus. The derivative tells how rapidly a quantity changes if we know how much of it there is at any instant. (If the change is at a constant rate, the rate of change is just the total change divided by the total time, and no derivative is needed.)

The second scenario is the opposite.

Scenario B

Your odometer is broken, but your speedometer works. Your passenger writes down the speed every second. How could you estimate the total distance covered?

This scenario is related to the “definite integral,” the key concept of integral calculus. This integral represents the total change in a varying quantity, if you know how rapidly it changes — even if the rate of change is not constant. (If the speed stays constant, you just multiply the speed times the total time, and no integral is needed.)

Both the derivative and the integral are based on limits, treated in Chapter 2. Chapter 3 defines the derivative, while Chapters 4 and 5 present some of its applications. Chapter 6 defines the integral.

As you would expect by comparing the two scenarios, the derivative and the integral are closely related. This connection is the basis of the Fundamental Theorem of Calculus (Section 6.4), which shows how the derivative provides a shortcut for computing many integrals.