Appendix A

Real Numbers

Summary

EXERCISES for Section A.0 *Key:* R–routine, M–moderate, C–challenging

1.[C]

2.[C]

Appendix B

Graphs and Lines

Exercises 3 to 9 concern solving simultaneous equations. By way of illustration we solve the equations

$$\begin{aligned} 2c_1 - 3c_2 &= 5 \\ 3c_1 + 4c_2 &= 6 \end{aligned}$$

in two different ways. In one approach we solve for one of the unknowns in terms of the other unknown (using one equation). Then we substitute the results in the other equation. Thus $c_1 = (5 + 3c_2)/s$, using the first equation. Substitution in the second equation gives $3(5 + 3c_2)/2 + 4c_2 = 6$, an equation in only one unknown. Solve it for $c - 2$, then get c_1.

In another approach we multiply each equation by a constant so that the coefficients of, say, c_1 become equal. Then subtract one equation from another. Thus

$$\begin{aligned} 3(2c_1 - 3c_2) &= 3 \cdot 5 \\ 2(3c_1 + 4c_2) &= 2 \cdot 6 \\ \text{or} \qquad 6c_1 - 9c_2 &= 15 \\ 6c_1 + 8c_2 &= 12. \end{aligned}$$

Subtracting gives $-17c_2 = 3$, hence $c_2 = \frac{3}{17}$. Then obtain c_1 using any of the equations. Both approaches apply to three equations in three unknowns.

Summary

EXERCISES for Section B.0 *Key:* R–routine, M–moderate, C–challenging

In Exercises 3 to 9 solve the simultaneous equations and check that your answers satisfy the equations.

3.[R]

$$\begin{array}{rrcl} 3c_1 & -2c_2 & = & 3 \\ c_1 & +c_2 & = & 4 \end{array}$$

4.[R]

$$\begin{array}{rrcl} 2c_1 & +5c_2 & = & -3 \\ 3c_1 & -4c_2 & = & 2 \end{array}$$

5.[R]

$$\begin{array}{rrcl} c_1 & +5c_2 & = & 6 \\ 2c_1 & -3c_2 & = & -2 \end{array}$$

6.[R]

$$\begin{array}{rrcl} 5c_1 & +2c_2 & = & 2 \\ -3c_1 & +4c_2 & = & 1 \end{array}$$

7.[R]

$$\begin{array}{rrrcl} c_1 & +2c_2 & +c_3 & = & 9 \\ c_1 & -c_2 & & = & -1 \\ c_1 & & +c_3 & = & 3 \end{array}$$

8.[R]

$$\begin{array}{rrrcl} 2c_1 & -c_2 & +c_3 & = & -7 \\ 3c_1 & -c_2 & -2c_3 & = & 5 \\ c_1 & +c_2 & +c_3 & = & -2 \end{array}$$

9.[R]

$$\begin{array}{rrrcl} c_1 & & +c_3 & = & 4 \\ & c_2 & -c_3 & = & -6 \\ c_1 & +c_2 & +c_3 & = & -1 \end{array}$$

Let $P(x)$ be a polynomial with integer coefficients. If $P(r) = 0$, then $x - r$ is a factor of $P(x)$. You may search for a root r by bisection method (Section 10.3) or

Newton's method (Section 10.4). There is an algebraic technique for determining any *rational* roots of $P(x) = 0$. Let $r = p/q$, where p and q are integers with no common divisor larger than 1. We may assume that q is positive. The rational root test asserts that if p/q is a root of $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$, then q must divide a_n and p must divide a_0.

For instance, consider $P(x) = 3x^3 + x^2 + x - 2$. If $P(p/q) = 0$, then p divides -2 and q divides 3. Then p must be 1, 2, or -2 and q must be 1 or 3. There are 8 combinations of p and q to check. For example, consider $p = 1$ and $q = 1$, that is $p/q = 1$. Note that $P(1) = 3$, so 1/1 is *not* a root. It turns out that the choice $p = 2$, $q = 3$ produces a root $\frac{2}{3}$. (Check that $P(2/3) = 0$.) Of course, a polynomial of degree greater than 1 need not have a rational root.

10.[R] Determine all rational roots of the following polynomials:

(a) $x^2 + x - 12$

(b) $2x^3 - 11x^2 + 17x - 6$

(c) $x^4 + x^3 + x^2 + x + 1$

(d) $3x^3 - 2x^2 - 4x - 1$

To factor a cubic $P(x) = ax^3+bx^2+cx+d$ first find or estimate a root r. Then divide $x-r$ into $P(x)$, obtaining a quotient $Q(x)$, that is, a quadratic polynomial such that $P(x) = Q(x)(x - r)$. If $Q(x)$ is reducible, factoring it completes the factorization of $P(x)$. If an integer is a root, it must divide the constant term (Why?). If a fraction m/n, when m and n are relatively prime integers, is a root, then m divides the constant term and n divides the coefficient of the highest power. (Why?) **11.**[R] Illustrate this procedure for

(a) $4x^3 + 4x^2 - 13x - 3$

(b) $2x^3 - x^2 - x - 3$

(c) $x^3 + x + 1$

(d) $x^3 - 8$

12.[R] Factor each of the following expressions:

(a) $x^3 - 2x^2 + x$

(b) $x^4 - 2x^2 + 1$

13.[M] This exercise outlines several ways to solve a system of simultaneous equations in several unknowns. You may recall learning a way to solve such systems using a determinant. This exercise presents an alternative.
Solve for A, B, and C.

$$\begin{cases} 2A & +B & +3C & = & 13 \\ 3A & +B & +2C & = & 11 \\ A & -B & +4C & = & 11 \end{cases}$$

(a) Subtract 2 times the second equation from 3 times the first equation. This gives an equation in just B and C. Solve for B in terms of C. Substitute this result into the second and third equations, which now involve only A and C. Now, solve these equations for A and C, then find B.

(b) As in (a), except solve the equation involving B and C for C in terms of B. Substitute this result into the second and third equations and proceed as in (a).

(c) First, subtract the second equation from the first, obtaining an equation in A and C. Then proceed as in (a).

(d) First, add the third equation to the second equation. Proceed as in (a).

In short, keep your eyes open for simplifications!

14.[C]

(a) In artithmetic, what is the analog of an irreducible polynomial?

(b) What is the analog of proper fractions of the partial fraction representation of proper rational rational functions? NOTE: By the way, mathematicians prove a single general theorem, which includes rational functions and rational numbers as special cases.

15.[C]

(a) In artithmetic, what is the analog of the partial fraction representation?

(b) What would it be for $\frac{34}{45}$?

SHERMAN: Insert Factor Theorem here. See also, Exercise 47.

16.[C] Prove that if a is a factor of the polynomial $P(x)$, then $P(a) = 0$.

Appendix C

Topics in Algebra

Summary

EXERCISES for Section C.0 *Key:* R–routine, M–moderate, C–challenging

17.[C]

18.[C]

Appendix D

Exponentials (and Logarithms)

This section focuses on exponentials; a general review of logarithms is presented in Section 1.5.

Summary

EXERCISES for Section D.0 *Key:* R–routine, M–moderate, C–challenging

19.[C]

20.[C]

Appendix E

Trigonometry

- discuss radian measure (§1.2)

Summary

EXERCISES for Section E.0 *Key:* R–routine, M–moderate, C–challenging

21.[C]

22.[C]

Appendix F

Logarithms and Exponentials Defined Through Calculus

Summary

EXERCISES for Section F.0 *Key:* R–routine, M–moderate, C–challenging

23.[C]

24.[C]

Appendix G

Determinants

Summary

EXERCISES for Section G.0 *Key:* R–routine, M–moderate, C–challenging

25.[C]

26.[C]

Appendix H

Jacobian and Change of Coordinates for Multiple Integrals

Summary

EXERCISES for Section H.0 *Key:* R–routine, M–moderate, C–challenging

27.[C]

28.[C]

Appendix I

Taylor Series for $f(x, y)$

Summary

EXERCISES for Section I.0 *Key:* R–routine, M–moderate, C–challenging

29.[C]

30.[C]

Appendix J

Parameterized Surfaces

Summary

EXERCISES for Section J.0 *Key:* R–routine, M–moderate, C–challenging

31.[C]

32.[C]

Appendix K

The Interchange of Limits

Summary

EXERCISES for Section K.0 *Key:* R–routine, M–moderate, C–challenging

33.[C]

34.[C]