Math 142 **Lab Project #4** **Due Mon., April 12 by 5:00 PM**

Assignment

Your fourth lab project consists of the three questions from the back of this page. Each question is worth 10 points.

Collaboration

You may work with others, but each student should submit a separately written project report. At the end of your report, you should list all of your collaborators. Simply copying another's work and calling it your own, or not giving credit to collaborators, is considered plagiarism and will be dealt with according to University regulations.

What makes a good report?

Your final report for each project should have a clear and complete presentation of the project material. Each report will generally consist of paragraphs of text interspersed between various Maple commands, plots, and other output. All text should be written with complete sentences using proper grammar and correct spelling. A part of your grade will be based upon the clarity of your presentation. When proofreading your report, check to see if you are omitting important details or illustrative plots. Also check to see if you are including too many unnecessary details. Hand in work that you are proud to claim as your own.

In particular, do not simply turn in a Maple worksheet with a bunch of input and output. It is not the TA's job to figure out how your Maple output answers the various questions. It is your job to submit an easy-to-read project report. You may use pencil and paper, Microsoft Word with bits of output copied from Maple, Maple with text regions, or any other format which allows you to submit a clearly written report. Note also that you are not required to use Maple for all parts of your project. Some work is more easily done by hand.

Submit project reports to your Lab TA (due Mon., April 12 by 5:00 PM)

Sections 1 – 3 with Prof. Dix
Lab TA: Qi Wu
Contact Info: wuq@math.sc.edu, LC 107B, 576-5948, mailbox for Wu, Qi by LC 411

Sections 4 – 6 with Mr. Murphy
Lab TA: Elizabeth Perez
Contact Info: pereze@math.sc.edu, LC107B, 576-5948, mailbox for Perez by LC 411

Sections 7 – 9 with Prof. Bennett
Lab TA: Luke Owens
Contact Info: owensl@math.sc.edu, LC B004, 777-4674, mailbox for Owens, L by LC 411

For problem 1, do not use Maple. For problems 2 and 3, it will be worthwhile to use the Maple program `AltHarmonicRearrange` introduced in this week's lab session.

1. (10 points)

   (a) Give a very clear explanation as to why the following two infinite series diverge to infinity.

   i. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{2k-1} = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \cdots$

   ii. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{2k} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \cdots$

   (b) Riemann proved the surprising result that a conditionally convergent series can be rearranged to converge to any given number. Problem #35 on page 457 of your textbook illustrates the reasoning behind Riemann's result and applies it to the alternating harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ which converges conditionally with a sum of $\ln 2$. Explain why the divergence of the two series in part (a) is essential to this line of reasoning.

2. (10 points) A rearrangement of the alternating harmonic series is formed with 50 positive terms, 2 negative terms, 50 positive terms, 2 negative terms, etc.

$$\underbrace{1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{99}}_{\text{50 pos. terms}} \underbrace{- \frac{1}{2} - \frac{1}{4}}_{\text{2 neg. terms}} \underbrace{+ \frac{1}{101} + \frac{1}{103} + \cdots + \frac{1}{199}}_{\text{50 pos. terms}} \underbrace{- \frac{1}{6} - \frac{1}{8}}_{\text{2 neg. terms}} + \cdots$$

   The sum of this rearranged series is given to be either $\ln 2$, $\ln 6$, $\ln 10$, or $\ln 14$. Based upon the first 520 partial sums of this series, which number is most likely the sum of this series? Sketch a graph which includes the points corresponding to the first 520 partial sums for this particular rearrangement as well as the line $y = L$ where L is the sum of this rearranged series.

3. (10 points) Using the method described in problem #35 on page 457 of your textbook, find a rearrangement of the alternating harmonic series which converges to π. Sketch a graph of the points corresponding to the first 475 partial sums for your rearrangement as well as the line $y = L$ where L is the sum of this rearranged series.

   Note: The last example from this week's Maple worksheet also illustrates the reasoning behind Riemann's result.