## Assignment

Your third lab project consists of the three questions from the back of this page. Each question is worth 10 points.

## Collaboration

You may work with others, but each student should submit a separately written project report. At the end of your report, you should list all of your collaborators. Simply copying another's work and calling it your own, or not giving credit to collaborators, is considered plagiarism and will be dealt with according to University regulations.

## What makes a good report?

Your final report for each project should have a clear and complete presentation of the project material. Each report will generally consist of paragraphs of text interspersed between various Maple commands, plots, and other output. All text should be written with complete sentences using proper grammar and correct spelling. A part of your grade will be based upon the clarity of your presentation. When proofreading your report, check to see if you are omitting important details or illustrative plots. Also check to see if you are including too many unnecessary details. Hand in work that you are proud to claim as your own.

In particular, do not simply turn in a Maple worksheet with a bunch of input and output. It is not the TA's job to figure out how your Maple output answers the various questions. It is your job to submit an easy-to-read project report. You may use pencil and paper, Microsoft Word with bits of output copied from Maple, Maple with text regions, or any other format which allows you to submit a clearly written report. Note also that you are not required to use Maple for all parts of your project. Some work is more easily done by hand.

## Submit project reports to your Lab TA (due Fri., March 26 by 5:00 PM)

Sections $1-3$ with Prof. Dix
Lab TA: Qi Wu
Contact Info: wuq@math.sc.edu, LC 107B, 576-5948, mailbox for Wu, Qi by LC 411
Sections $4-6$ with Mr. Murphy
Lab TA: Elizabeth Perez
Contact Info: pereze@math.sc.edu, LC107B, 576-5948, mailbox for Perez by LC 411
Sections $7-9$ with Prof. Bennett
Lab TA: Luke Owens
Contact Info: owensl@math.sc.edu, LC B004, 777-4674, mailbox for Owens, L by LC 411

## Probability

## $\underline{\text { Definitions and Terminology }}$

- A random variable is a quantity that can be measured or determined but might vary each time the experiment is repeated. For example, if the experiment is flipping a coin until a head appears, the number of flips before seeing a head is a (discrete) random variable. The number of hours that a lightbulb burns before failing is an example of a (continuous) random variable.
- A probability density function, or $p d f$, for a continuous random variable $X$ is a function $f$ defined on $(-\infty, \infty)$ with the properties
(1) $f(x) \geq 0$ for all $-\infty<x<\infty$
(2) $\int_{-\infty}^{\infty} f(x) d x=1$
- The probability that $X$ takes on a value between $a$ and $b$ is

$$
\operatorname{Prob}(a \leq X \leq b)=\int_{a}^{b} f(x) d x
$$

- The mean, or expected value, of a continous random variable $X$ with pdf $f$ is defined to be

$$
\mu=\mathrm{E}[X]=\int_{-\infty}^{\infty} x f(x) d x
$$

provided this improper integral exists.

- The variance of $X$ is defined to be

$$
\sigma^{2}=\operatorname{Var}[X]=\mathrm{E}\left[(X-\mu)^{2}\right]=\int_{-\infty}^{\infty}(x-\mu)^{2} f(x) d x
$$

provided this improper integral exists.
Note: $\sigma$ is called the standard deviation of $X$.

- The median value of $X$ is defined to be the number $m$ such that

$$
\int_{-\infty}^{m} f(x) d x=\frac{1}{2} .
$$

- The cumulative distribution function, or $c d f$, for $X$ is the function $F$ with the property that

$$
F(x)=\operatorname{Prob}[X \leq x]=\int_{-\infty}^{x} f(t) d t
$$

where $f$ is the $p d f$ for $X$.

## Problems

(1) Which of the following are probability density functions. Explain.
(a) $f(x)= \begin{cases}\frac{1}{3}(x-1)(3 x-7) & \text { if } 0 \leq x \leq 3 \\ 0 & \text { if } x<0 \text { or } x>3\end{cases}$
(b) $f(x)= \begin{cases}4 x-2 x^{2} & \text { if } 0 \leq x \leq 2 \\ 0 & \text { if } x<0 \text { or } x>2\end{cases}$
(c) $f(x)=\frac{1}{\sqrt{2 \pi}} e^{-x^{2} / 2}$
(2) Determine the value of $C$ which will make $f(x)=\frac{C}{1+x^{2}}$ a probability density function. Using this value of $C$, obtain a formula for the cumulative distribution function $F(x)$ which corresponds to this pdf. Now sketch graphs of $f(x)$ and $F(x)$ together on the same set of axes.
(3) The probability density function for the number of miles that a car can run before its newly installed battery wears out is

$$
f(x)= \begin{cases}0.1 e^{-0.1 x} & \text { if } x \geq 0 \\ 0 & \text { if } x<0\end{cases}
$$

where $x$ is measured in thousands of miles.
(a) Sketch the graph of this probability density function.
(b) What is the probability that a newly installed battery will wear out at some point between the car being driven 1000 miles and 3000 miles?
(c) Suppose you've just installed a new battery in your car and would like to take a 5000-mile trip. What is the probability that you will be able to complete your trip before your car battery wears out?
(d) What is the mean number of miles that a newly installed car battery will last before wearing out?
(e) What is the median number of miles that a newly installed car battery will last before wearing out? Explain the geometric significance of this value in terms of areas under the graph of $f(x)$.
(f) If the probability that a newly installed battery wears out in less than $T$ miles is 0.75 , then what is the value of $T$ ?

