Differentiation of Transcendental Functions

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Overview

This lab uses Maple to help you practice your differentiation skills. Three different user interfaces are introduced.

- The *Derivatives* tutor takes a user-specified function, finds its derivative and plots the function and its derivative on a specified domain.

- The *Differentiation Methods* tutor shows the step-by-step application of the differentiation rules involved in finding the derivative of a user-specified function.

- The *DerivativeDrill* maplet allows the user to select types of functions and rules and then generates random problems of this type; the user has to enter the correct derivative. This maplet also provides practice entering mathematical expressions in Maple syntax.

Used properly, these are excellent tools for helping students hone their understanding of derivatives and their differentiation skills.

Maple Essentials

- The new Maple commands introduced in this lab are:

Command	Description
`exp`	exponential function, e^x
`ln`	(natural) logarithm function, $\ln x$
`log`	(natural) logarithm function, $\ln x$
`log[10]`	base 10 logarithm function, $\log_{10} x$

- The *Derivatives* tutor is started from the Maple 9.5 user interface under the Tools menu:

  Tools → Tutors → Calculus - Single Variable → Derivatives ...

  *This tutor plots a given function and its derivative. Be sure to specify a reasonable domain by entering appropriate values in the boxes labelled $a =$ and $b =$. The **Display** button displays the formula for the derivative and plots the function and derivative on the specified domain. Look at the plot to see how the derivative can be used to determine where the function is increasing, decreasing, concave up, and concave down. This is a great tool to use when you need to check your answer to a problem in the text.*

- The *Differentiation Methods* tutor is started from the Maple 9.5 user interface under the Tools menu:

  Tools → Tutors → Calculus - Single Variable → Differentiation Methods ...

  *This tutor is designed to help you practice the individual steps involved in finding the derivative of a given function. Do not be afraid to use the **Hint** and **Apply Hint** — particularly when the hint suggests using the Rewrite rule. Also, you might find it useful to mark the Constant, Constant Multiple, and Identity rules as being understood (see the **Understood Rules** menu).*

- The *DerivativeDrill* maplet is available from USC at the URL:

  `http://www.math.sc.edu/~meade/141L-F04/maplets/CalcUSC/DerivativeDrill.maplet`

  This maplet generates random differentiation problems using the types of functions and differentiation rules that you select and checks if you correctly find the derivative. Start slowly and build to more complicated expressions as you gain confidence in your differentiation skills.

Preparation

Prior to the beginning of this lab, be sure you know the basic differentiation rules (including implicit differentiation).

Activities

- Here are a few extended questions to get you started.

  1. Use the `DerivativeDrill` maplet with polynomial, trigonometric, exponential, and logarithmic functions that utilize the power, product, quotient, chain rules. Start with only one rule per problem. As you gain confidence, allow up to 2 rules in each problem.

  2. For each of the functions listed below, find the derivative and plot the function and derivative for $-3 \leq x \leq 3$.
     (a) $y = 3^x$
     (b) $y = 2^x$
     (c) $y = e^x$
     (d) $y = \pi^x$
     How are these four functions related to their derivatives?

  3. For each of the functions listed below, find the derivative and plot the function and derivative for $0 < x \leq A$ for an appropriate choice of A.
     (a) $y = \ln\left(\dfrac{1}{x}\right)$
     (b) $y = \dfrac{1}{\ln x}$ (where is the vertical asymptote?)
     (c) $y = \ln\left(x^2\right)$
     (d) $y = (\ln x)^2$
     (e) $y = \dfrac{\ln x}{x}$.
     (f) $y = x \ln x$.

  4. List the rules involved in finding the derivative of each of the following:
     (a) $y = x^\pi$
     (b) $y = \pi^x$
     (c) $y = x^x$
     (d) $y = \pi^\pi$.

  5. Let $F(x) = x^3 + 3x^2 + 7$ and $G(x) = \tan(x^2 + 1)$. Find the derivatives of $F(x)G(x)$, $\dfrac{F(x)}{G(x)}$, and $(F \circ G)(x)$.
     Suppose F and G have been defined as Maple expressions. The composition, $F \circ G$, can be obtained with `FoG := eval( F, x=G );`.

  6. Find the first five derivatives of $y = xe^x$. What is $\dfrac{d^{100}y}{dx^{100}}$?

Assignment

There is no assignment for this lab.

Any calculus textbook will have hundreds more differentiation exercises. In addition, use your imagination to create your own examples.

Remember to use the ideas presented in this lab to help you check derivatives that you compute manually.