

# Max/Min and Graphing Problems

**Objective** This lab requires the use of the First and Second Derivative Tests to perform qualitative analysis for a function and to answer applied optimization problems.

**Background** Maple can be used to create a reasonable graph of almost any function that we are likely to encounter in this course. Because of this, the “graphing” problems in this lab do not ask you to create a graph of the function. Instead, you are requested to provide accurate qualitative information about the function: intervals on which the function is increasing/decreasing and concave up/down, local and global extrema, and inflection points.

This information can be obtained from the first and second derivatives of a function. The `plot` command with an appropriate viewing window can create a reasonable graph. The graph can typically be used to estimate intercepts, critical points, and possible inflection points. You are already familiar with the use of the `fsolve` command to obtain floating-point approximations to solutions. Recall that when an equation has more than one solution it can be difficult to know which solution `fsolve` would return. A slight modification of the syntax for `fsolve` forces `fsolve` to return a solution in a specific interval. You should get in the habit of using a plot to determine an interval where the equation has exactly one solution and then use `fsolve` to approximate that specific solution.

To apply the First or Second Derivative Test, find the stationary points using `solve` and/or `fsolve`. A graph can provide preliminary estimates for these values. If you see that  $f'(x) = 0$  for exactly one value of  $x$  between  $a$  and  $b$ , the command `fsolve( Df(x)=0, x=a..b );` will return this solution.

The `FunctionChart` command from the `Student[Calculus1]` package can be used to create well-labeled plots of a function and its principle qualitative properties.

**Discussion** Enter, and execute, the following Maple commands in a Maple worksheet.

## Example 1: Initial Analysis

```
> restart; # clear Maple's memory
> with( plots ); # load package
> f := x->( -x^3 + 3*x^2 - 5*x + 6 ) # define rational function
>      /( x^2 - 2*x - 3 );
> p0 := plot( f(x), x=-10..15, y=-20..20, # create plot
>      discontinuous=true );
> p0; # display plot of function
> Xint := [ 0, f(0) ]; # x-intercept
> q1 := fsolve( f(x)=0, x=1..3 ); # y-int in [1,3]
> Yint := [ q1, f(q1) ]; # y-intercept
```

## Example 2: Analysis of First Derivative

```
> Df := D( f ); # 1st derivative
> p1 := plot( Df(x), x=-10..15, y=-20..20, # create plot
>      discontinuous=true, color=blue );
> p1; # display plot of 1st deriv
> q2 := fsolve( Df(x)=0, x=-4..-2 ); # sol to f'(x) = 0 in [-4,-2]
> CP1 := [ q2, f(q2) ]; # crit point in [-4,-2]
> q3 := fsolve( Df(x)=0, x=4..5 ); # sol to f'(x) = 0 in [4,5]
> CP2 := [ q3, f(q3) ]; # crit point in [4,5]
```

### Example 3: Analysis of Second Derivative

```
> D2f := D( Df ); # 2nd derivative
> p2 := plot( D2f(x), x=-10..15, y=-20..20, # create plot
>         discont=true, color=green );
> p2; # display plot of 2nd deriv
> q4 := fsolve( D2f(x)=0, x= 1.. 2 ); # possible infl pt in [ 1, 2]
> IP := [ q4, f(q4) ]; # change conc  $\implies$  infl pt
> display( [p0,p1,p2] ); # plot of  $f, f', f''$ 
> p3 := pointplot( [Xint,Yint,CP1,CP2,IP], # plot of interesting pts
>         symbolsize=18 );
> display( [p0,p3] ); # function w/interest pts
> D2f(q2); # concavity @ crit pt
> D2f(q3); # concavity @ crit pt
```

### Example 4: Using the FunctionChart command

```
> with( Student[Calculus1] );
> f := x->( -x^3 + 3*x^2 - 5*x + 6 )/( x^2 - 4*x + 3 );
> FunctionChart( f(x), x=-10..15, view=[DEFAULT,-20..20] );
> FunctionChart( f(x), x=-10..15, view=[DEFAULT,-20..20]
>         pointoptions=[symbolsize=18],
>         slope=[thickness(2,2), color(red,blue)],
>         concavity=[filled(yellow,green)] );
```

## Notes

- (1) Use `?FunctionChart` to see a complete list of options for the `FunctionChart` command.
- (2) For a general equation, `fsolve` returns only one solution. But, if the equation is a polynomial, `fsolve` returns all real-valued solutions. Compare `solve( Df(x)=0, {x} )`, `fsolve( Df(x)=0, x )`, and `fsolve( numer(normal(Df(x)))=0, x )`.
- (3) To use a Greek letter as a variable name in Maple, use the English spelling of the letter and Maple will display the name in Greek. See `?greek` for the full list of Greek letters.
- (4) The `CurveAnalysis` and `FunctionAnalyzer` maplets are designed to assist with the qualitative analysis of a function. Links to these maplets can be found on the WWW at <http://www.math.sc.edu/~meade/141L-F03/maplets/>.

## Questions

- (1) Let  $g(t) = \frac{t\sqrt{t-3}}{(t-2)^2}$ . Use the First and/or Second Derivative Tests to identify all local and global extrema and inflection points on the graph of  $y = g(t)$ .
- (2) Let  $f(\theta) = \sin(\theta) - \sin^2(\theta)$ . Use the First and/or Second Derivative Tests to identify all local and global extrema and inflection points on the graph of  $y = f(\theta)$  on the interval  $[-\pi, \pi]$ .
- (3) An observatory is to be constructed in the form of a right circular cylinder topped with a hemispherical dome. If the hemispherical dome costs twice as much per square foot as the cylindrical wall (and there is no cost for flooring), what are the most economical dimensions for a given volume? (Your answer should express the radius and height in terms of the volume,  $V$ , and price,  $p$ , per square foot for the cylindrical wall.) What is the total cost to build an observatory with volume  $V = 8000$  cubic feet and the cheaper material costs \$12 per square foot.