

ALGEBRA PROBLEMS SET TWENTY AND A HALF
DUE 14 APRIL 2015

PROBLEM 0. Let \mathbf{H} and \mathbf{N} be normal subgroups of the group \mathbf{G} . Prove that $[\mathbf{H}, \mathbf{N}] = [\mathbf{N}, \mathbf{H}]$.

PROBLEM 1. Let \mathbf{N} be a normal subgroup of the group \mathbf{G} . Prove that \mathbf{G} is solvable if and only if both \mathbf{N} and \mathbf{G}/\mathbf{N} are solvable.

PROBLEM 2. Show that a finite group has a unique largest solvable normal subgroup.

PROBLEM 3. Let p and q be prime numbers with $p > q$ and let n be any natural number. Prove that every group of order $p^n q$ is solvable. [Hint: Ask Sylow to help you.]