NAME: DIFFY QUEUE

SAMPLE FINAL EXAMINATION
MATH 242 SECTION H01
2014

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REMEMBER TO SHOW ALL YOUR WORK!

Problem 0 (Core)
Find solutions to each of the following initial value problems.

(a) $\begin{aligned} xy' + 3y &= 2x^5 \\ y(2) &= 1 \end{aligned}$ (b) $\begin{aligned} y' &= 6e^{2x-y} \\ y(0) &= 0 \end{aligned}$

Problem 1
In each part below determine whether the functions listed are linearly independent or linearly dependent. Be sure to explain your reasoning fully.

(a) $1 - x$ and $1 + |x|$ (b) $\cos^2 x$ and $1 + \cos 2x$

Problem 2
In each part below, find two linearly independent solutions to the given differential equation.

(a) $y'' + 6y' + 8y = 0$ (b) $y'' + 6y' + 9y = 0$ (c) $y'' + 6y' + 25y = 0$

Problem 3 (Core)
Solve each of the initial value problems below.

(a) $\begin{aligned} y'' - 4y &= 0 \\ y(0) &= 1 \\ y'(0) &= 2 \end{aligned}$ (b) $\begin{aligned} y'' + 4y &= 0 \\ y(0) &= 1 \\ y'(0) &= 2 \end{aligned}$

Problem 4

Find the general solution to

$$y'' + 2y' + 5y = e^x \sin x$$

Problem 5 (Core)
Find the general solution for

$$y'' + 4y = \sin^2 x$$

Problem 6
A mass of 1kg is attached to a spring with constant $k = 4$kg/m. Initially, the system is at equilibrium (that is $x(0) = 0$ and $x'(0) = 0$). But for all times $t > 0$ the mass is subject to a periodic external force $f(t) = \sin 3t$. Find the resulting motion $x(t)$.

Problem 7 (Core)
Consider a system consisting of a mass of 2 kg attached to a spring with constant of 1 kg/sec^2 and subject to a frictional force proportiional to the velocity with a constant of 2kg/sec. Initially the system is at equilibrium (that is, $x(0) = 0$ and $x'(0) = 0$), but for $t > 0$ the mass is subject to a external force of $5 \cos t$. Find the motion $x(t)$ and identify the steady-periodic part and the transient part.x

Problem 8 (Core)
Find the Laplace transform of each function below.

$$\text{(a)} \quad f(t) = \sin 2t \cos 2t \qquad\qquad \text{(b)} \quad g(t) = t\cos t + te^{2t}.$$

Problem 9
For each part below find the function whose Laplace transform is given.

$$\text{(a)} \quad \mathcal{L}[f(t)] = \frac{1}{s^2(s^2-1)} \qquad \text{(b)} \quad \mathcal{L}[g(t)] = \frac{2s+1}{s(s^2+9)}$$

Problem 10 (Core)
Use the method of Laplace transforms to find the solution to the following initial value problem.

$$y'' + 4y' + 8y = e^{-t} \qquad y(0) = 0 \qquad y'(0) = 0$$

Problem 11
Solve the following initial value problem by the method of your choice.

$$y'' + 4y' + 5y = 39e^t \sin t \qquad y(0) = -1 \qquad y'(0) = -1$$