Quiz #10, Math 115

Dec 1st, 2006

Name:________

Direction: Please *print* your name. And *show your work for credit.*
You may use the following identities:
$\sin(x+y) = \sin x\cos y + \cos x \sin y$, $\sin(x-y) = \sin x \cos y - \cos x \sin y$
$\cos(x+y) = \cos x \cos y - \sin x \sin y$, $\cos(x-y) = \cos x \cos y + \sin x \sin y$
$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$, $\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}$

1. Find the exact values of the following:

$$\sin 15^\circ \quad \cos 75^\circ \quad \tan 75^\circ$$

$$\sin(15^\circ) = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos(75^\circ) = \cos(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\tan(75^\circ) = \tan(45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$
$$= \frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)}$$
$$= 2 - \sqrt{3}$$

2. If $0 < x < \pi$, $\sin x = \frac{2}{3}$ and $\cos x$ is negative. Find the following:

$$\sin 2x \quad \cos\frac{x}{2}$$

Hint: First, use Pythagorean Identity.

$$\sin^2 x + \cos^2 x = 1$$
$$\left(\frac{2}{3}\right)^2 + \cos^2 x = 1$$
$$\cos^2 x = \frac{5}{9} \quad \text{and } \cos x \text{ is negative}$$
$$\Rightarrow \cos x = -\frac{\sqrt{5}}{3}$$

$$\sin 2x = 1 - 2\sin^2 x = 1 - 2\cdot\left(\frac{2}{3}\right)^2 = \frac{1}{9}$$

$\cos\frac{x}{2}$ is pos. since $0 < x < \pi$ (i.e. $0 < \frac{x}{2} < \frac{\pi}{2}$)

$$\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\frac{\sqrt{5}}{3}}{2}} = \sqrt{\frac{3-\sqrt{5}}{6}}$$

3. Identity verification:

$$\frac{1-\cos^2 y}{(1-\sin y)(1+\sin y)} = \tan^2 y$$

LHS: $\dfrac{1-\cos^2 y}{(1-\sin y)(1+\sin y)} = \dfrac{\sin^2 y}{1-\sin^2 y}$ ← Pythagorean Identity ← Special product

$$= \frac{\sin^2 y}{\cos^2 y}$$

$$= \left(\frac{\sin y}{\cos y}\right)^2$$

$$= \tan^2 y$$