

22. Find all solutions of $6x - 7\sqrt{x} - 3 = 0$ The first step is moving $7\sqrt{x}$ to right side. Thus, we have $6x - 3 = 7\sqrt{x}$. Square both sides, we get $(6x - 3)^2 = 36x^2 - 36x + 9$ and $(7\sqrt{x})^2 = 49x$. So, $36x^2 - 36x + 9 = 49x$ and hence $36x^2 - 85x + 9 = 0$. Now factor $36x^2 - 85x + 9 = 0$ as $(4x - 9)(9x - 1) = 0$. This means $x = \frac{1}{9}$ and $x = \frac{9}{4}$ are possible solutions. However, when plugging the two answers in the original equation, we find that $\frac{1}{9}$ is an extraneous solution. Therefore the answer of the equations is $x = \frac{9}{4}$.

Another way to solve this equation is using substitution. Let $\sqrt{x} = u$, so $x = u^2$ by squaring both sides. Therefore, $6u^2 - 7u - 3 = 0$. Factor it, we get $(3u - 1)(2u - 3) = 0$. Hence $u = \frac{1}{3}$ or $u = \frac{3}{2}$. We should be very careful since we want to solve x not u , so the final answer should be $x = u^2 = \frac{1}{9}$ or $x = u^2 = \frac{9}{4}$. Again, the former one is extraneous.

46. Solve $(x + 3)^{\frac{3}{2}} = 8$. Firstly, square both sides then we get $(x + 3)^3 = [(x + 3)^{\frac{3}{2}}]^2 = (8)^2 = 64$. Now, $(x + 3)^3 = 64 = 4^3$ which means $x + 3 = 4$. Immediately, we get $x = 1$.
58. Solve the rational equation $\frac{4}{x} - \frac{5}{3} = \frac{x}{6}$. Because both sides contain several fractions, we have to multiply all fractions by their common multiple. The least common multiple of $x, 3$ and 6 is $6x$. So, we multiply the both sides of the equation by $6x$. $(\frac{4}{x} - \frac{5}{3})(6x) = \frac{x}{6}(6x)$, simplify it, we have $24 - 10x = x^2$. Furthermore, we can convert it as $0 = x^2 + 10x - 24 = (x + 12)(x - 2)$. Thus, 2 and -12 are the answers and it is not hard to check that none of them is extraneous.
66. Solve $|3x + 2| = 7$. To take off the absolute value sign, we have to write it as $3x + 2 = \pm 7$. So, we have to solve $3x + 2 = 7$ and also solve $3x + 2 = -7$, two equations. Therefore, either $x = \frac{5}{3}$ or $x = -3$.