$H_z := -\text{LambertW}\left(-\frac{1}{2}\exp\left(\frac{1}{2}\cdot z - \frac{1}{2}\right)\right) + \frac{1}{2}\cdot z - \frac{1}{2};$

$$-\text{LambertW}\left(-\frac{1}{2}\, e^{\frac{1}{2} z - \frac{1}{2}}\right) + \frac{1}{2} z - \frac{1}{2} \quad \textbf{(1)}$$

$H_s := subs\left(z = (-1 + 2\cdot\ln(2))\cdot(1 - \Delta^2), H_z\right);$

$$-\text{LambertW}\left(-\frac{1}{2}\, e^{\frac{1}{2}(-1+2\ln(2))(1-\Delta^2) - \frac{1}{2}}\right) + \frac{1}{2}(-1+2\ln(2))(1-\Delta^2) - \frac{1}{2} \quad \textbf{(2)}$$

$\text{rho} := -1 + 2\cdot\ln(2);$

$$-1 + 2\ln(2) \quad \textbf{(3)}$$

$H_{sing} := map\left(simplify, series\left(H_s, \text{Delta} = 0, 10\right)\right); \text{Delta} = \text{sqrt}(\text{``}(1 - z/\text{rho}));$

$$\ln(2) - \frac{1}{2}\sqrt{2}\sqrt{-2+4\ln(2)}\,\Delta + \left(\frac{1}{6} - \frac{1}{3}\ln(2)\right)\Delta^2 - \frac{1}{72}\sqrt{2}\sqrt{-2+4\ln(2)}\,(-1 + 2\ln(2))\,\Delta^3 - \frac{1}{270}(-1+2\ln(2))^2\Delta^4 - \frac{1}{8640}\sqrt{2}\sqrt{-2+4\ln(2)}\left(1 - 4\ln(2) + 4\ln(2)^2\right)\Delta^5 + \frac{1}{17010}(-1+2\ln(2))^3\Delta^6 + \frac{139}{10886400}\sqrt{2}\sqrt{-2+4\ln(2)}\left(-1 + 6\ln(2) - 12\ln(2)^2 + 8\ln(2)^3\right)\Delta^7 + \frac{1}{204120}(-1+2\ln(2))^4\Delta^8 + \frac{571}{4702924800}\sqrt{2}\sqrt{-2+4\ln(2)}\left(1 - 8\ln(2) + 24\ln(2)^2 - 32\ln(2)^3 + 16\ln(2)^4\right)\Delta^9 + O\left(\Delta^{10}\right)$$

$$\Delta = \sqrt{\left(1 - \frac{z}{-1+2\ln(2)}\right)} \quad \textbf{(4)}$$

$H_{asympt} := n!\cdot asympt\left(coeff\left(H_{sing}, \text{Delta}, 1\right)\cdot \text{rho}\,\hat{}\,(-n)\cdot subs(\{\cos(\text{Pi}\cdot n) = 1, \text{O} = 0\}, simplify(asympt(\text{binomial}(1/2, n), n, 2))), n\right);$

$$\frac{1}{4}\,\frac{n!\,\sqrt{2}\sqrt{-2+4\ln(2)}\left(\frac{1}{n}\right)^{3/2}}{\sqrt{\pi}\,(-1+2\ln(2))^n} \quad \textbf{(5)}$$

$H_{asymptexpansion} := n!\cdot asympt\left(coeff\left(H_{sing}, \text{Delta}, 1\right)\cdot \text{rho}\,\hat{}\,(-n)\cdot subs(\{\cos(\text{Pi}\cdot n) = 1\}, simplify(asympt(\text{binomial}(1/2, n), n, 4))), n, 8\right);$

$$\frac{1}{(-1+2\ln(2))^n}\left(n!\left(\frac{1}{4}\,\frac{\sqrt{2}\sqrt{-2+4\ln(2)}\left(\frac{1}{n}\right)^{3/2}}{\sqrt{\pi}}\right.\right. \quad \textbf{(6)}$$

$$+\frac{3}{32}\frac{\sqrt{2}\sqrt{-2+4\ln(2)}\left(\frac{1}{n}\right)^{5/2}}{\sqrt{\pi}}+\frac{25}{512}\frac{\sqrt{2}\sqrt{-2+4\ln(2)}\left(\frac{1}{n}\right)^{7/2}}{\sqrt{\pi}}$$

$$+\mathrm{O}\left(\left(\frac{1}{n}\right)^{9/2}\right)\Bigg)\Bigg)$$

$A := unapply(\,\mathbf{(6)}, n\,);$

$$n \to \frac{1}{(-1+2\ln(2))^{n}}\Bigg(n!\Bigg(\frac{1}{4}\frac{\sqrt{2}\sqrt{-2+4\ln(2)}\left(\frac{1}{n}\right)^{3/2}}{\sqrt{\pi}}$$

$$+\frac{3}{32}\frac{\sqrt{2}\sqrt{-2+4\ln(2)}\left(\frac{1}{n}\right)^{5/2}}{\sqrt{\pi}}+\frac{25}{512}\frac{\sqrt{2}\sqrt{-2+4\ln(2)}\left(\frac{1}{n}\right)^{7/2}}{\sqrt{\pi}}$$

$$+\mathrm{O}\left(\left(\frac{1}{n}\right)^{9/2}\right)\Bigg)\Bigg) \quad \mathbf{(7)}$$

$expect := simplify\left(asympt\left(\frac{A(n+2)}{2\cdot A(n+1)}-\frac{n+1}{2}, n, 5\right)\right);$

$$\frac{1}{4}\frac{4n-4n\ln(2)+3-4\ln(2)-4\,\mathrm{O}\left(\frac{1}{n}\right)+8\,\mathrm{O}\left(\frac{1}{n}\right)\ln(2)}{-1+2\ln(2)} \quad \mathbf{(8)}$$

$dsquare = simplify\left(asympt\left(\frac{A(n+3)}{4\cdot A(n+1)}-\frac{A(n+2)^2}{4\cdot A(n+1)^2}-\frac{A(n+2)}{2\cdot A(n+1)}-\frac{n+1}{4}, n, 7\right)\right);$

$$dsquare=\frac{1}{8}\frac{1}{(-1+2\ln(2))^2}\Big(4n-8n\ln(2)^2+1+4\ln(2)-8\ln(2)^2+8\,\mathrm{O}\left(\frac{1}{n}\right)$$

$$-32\,\mathrm{O}\left(\frac{1}{n}\right)\ln(2)+32\,\mathrm{O}\left(\frac{1}{n}\right)\ln(2)^2\Big) \quad \mathbf{(9)}$$