

COURSE ANNOUNCEMENT

Course Title	Math 747 Algebraic Geometry
Semester	Spring 2009
Time	MWF, 12:20PM- 1:10PM
Instructor	Kustin
Textbook:	Introduction to Algebraic Geometry by Brendan Hassett Cambridge University Press, 2007
The Course Grade:	will be based on Homework

The course will closely follow the text book. The topics that will be covered in the course are the same as the Chapter titles:

1. Guiding problems (Implicitization, Ideal Membership, Interpolation)
2. Division algorithm and Gröbner bases
3. Affine varieties
4. Elimination
5. Resultants
6. Irreducible varieties
7. Nullstellensatz
8. Primary decomposition
9. Projective geometry
10. Projective elimination theory
11. Parametrizing linear subspaces
12. Hilbert polynomials and the Bezout Theorem

We will start at the beginning and proceed in order. Each Chapter contains many exercises (14 in Chapter 1, 18 in Chapter 2, 25 in Chapter 3, etc.) The exercises often involve a computation and all of them are down-to-earth.

The best way to convey the plan of the course is to copy the author's preface. This preface exactly describes the intentions of the instructor of this course. "This book is an introduction to algebraic geometry, based on courses given at Rice University and the Institute of Mathematical Sciences of the Chinese University of Hong Kong from 2001 to 2006. The audience for these lectures was quite diverse, ranging from second-year undergraduate students to senior professors in fields like geometric modeling or differential geometry. Thus the algebraic prerequisites are kept to a minimum: a good working knowledge of linear algebra is crucial, along with some familiarity with basic concepts from abstract algebra. A semester of

formal training in abstract algebra is more than enough, provided it touches on rings, ideals, and factorization. In practice, motivated students managed to learn the necessary algebra as they went along.

There are two overlapping and intertwining paths to understanding algebraic geometry. The first leads through sheaf theory, cohomology, derived functors and categories, and abstract commutative algebra – and these are just the prerequisites! We will not take this path. Rather, we will focus on specific examples and limit the formalism to what we need for these examples. Indeed, we will emphasize the strand of formalism most useful for computations: We introduce *Gröbner bases* early on and develop algorithms for almost every technique we describe. The development of algebraic geometry since the mid 1990s vindicates this approach. The term ‘Groebner’ occurs in 1053 Math Reviews from 1995 to 2004, with most of these occurring in the last five years. The development of computers fast enough to do significant symbolic computations has had a profound influence on research in the field.

A word about what this book will *not* do: We develop computational techniques as a means to the end of learning algebraic geometry. However we will not dwell on the technical questions of computability that might interest a computer scientist. We also will not spend time introducing the syntax of any particular computer algebra system. However it is necessary that the reader be willing to carry out involved computations using elementary algebra, preferably with the help of a computer algebra system such as *Maple*, *Macaulay II*, or *Singular*.

Our broader goal is to display the core techniques of algebraic geometry in their natural habitat. These are developed systematically, with the necessary commutative algebra integrated with the geometry. Classical topics like resultants and elimination theory, are discussed in parallel with affine varieties, morphisms, and rational maps. Important examples of projective varieties (Grassmannians, Veronese varieties, Segre varieties) are emphasized, along with the matrix and exterior algebra needed to write down their defining equations.

It must be said that this book is not a comprehensive introduction to all of algebraic geometry. Shafarevich’s book comes closest to this ideal; it addresses many important issues we leave untouched. Most other standard texts develop the material from a specific point of view, e.g., sheaf cohomology and schemes (Hartshorne), classical geometry (Harris), complex algebraic differential geometry (Griffiths and Harris), or algebraic curves (Fulton).”