Homework 4

Due at the beginning of class on Wednesday, Feb. 18

(14) I found the following procedure on the Internet at
`http://www.math.uiuc.edu/Macaulay2/doc/Macaulay2-1.1.99/share/Macaulay2/Macaulay2Doc/groebner.m2`

Apparently this procedure will write any symmetric polynomial k[x,y,z] as a polynomial in the elementary symmetric polynomials. I want you to explain what this procedure is doing and why it works.

document

Key => "normal forms",

TEX ///Let R = $k[x_1, ..., x_n]$ be a polynomial ring over a field k,

and let $I \subset R$ be an ideal. Let $\{g_1, ..., g_t\}$ be a Groebner

basis for I. For any $f \in R$, there is a unique 'remainder' $r \in R$ such

that no term of r is divisible by the leading term of any g_i and such

that $f - r$ belongs to I. This polynomial r is sometimes called the normal

form of f.///,

PARA,

"For an example, consider symmetric polynomials. The normal form of the

symmetric polynomial ", TT "f", " with respect

to the ideal ", TT "I", "below writes ", TT "f", " in terms of the elementary symmetric functions ", TT "a,b,c", ".",

EXAMPLE lines ///

R = QQ[x,y,z,a,b,c,MonomialOrder=>Eliminate 3];

I = ideal(a-(x+y+z), b-(x*y+x*z+y*z), c-x*y*z)

f = $x^3 + y^3 + z^3$

f % I

///,

SeeAlso => "Grbner bases", (symbol %, RingElement, Ideal),

(15) On page 43, Hassett sends us to Shafarevich for a problem: Prove that the map $f(x, y) = (\alpha x, \beta y + P(x))$ is an automorphism of $\mathbb{A}^2$, where α and β are nonzero elements of k and $P(x)$ is a polynomial. Prove that maps of this type form a group.

(16) Hassett page 55, problem 3.14.